Superconvergent Interpolants for Collocation Solutions of Mixed First and Second Order Boundary Value ODE Systems

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The high quality COLSYS/COLNEW Gaussian collocation software package has been widely used for the numerical solution of boundary value ordinary differential equations (BVODEs) for approximately four decades and, more recently, interfaces to the package have been developed for several high level problem solving nvironments/scripting languages such as Scilab, R, and Python. The collocation algorithm employs a mesh that partitions the domain of the BVODE and it is well-known that the computed continuous collocation solution has higher accuracy, i.e., is superconvergent, at the meshpoints. For systems of BVODEs represented as first order systems, it has been shown that an approach based on continuous Runge-Kutta (CRK) methods can be employed to obtain an efficiently implementable superconvergent interpolant (SCI) across the problem domain. Recent work has seen the development of a new version of COLNEW, called COLNEWSC, that computes and returns an error controlled SCI, leading to substantial efficiency improvements.

However, it is common for BVODEs to include higher derivatives and a feature of the COLSYS/COLNEW package is that it can directly handle mixed order BVODE systems. In this report we show that it is possible to extend the approach based on the use of CRK methods to obtain SCIs for collocation solutions of mixed first and second order BVODE systems. In addition to identifying a general framework for the study of these methods, based on continuous Runge-Kutta-Nystr\"om methods, we derive specific methods that yield SCIs for mixed first and second order BVODE systems having orders of accuracy equal to 2, 4, 6, and 8. We also provide numerical results to verify the orders of convergence of the SCIs that we derive.