

Saint Mary’s University

DEPARTMENT OF MATHEMATICS  
AND COMPUTING SCIENCE

Name: \_\_\_\_\_

Signature: \_\_\_\_\_

ID: \_\_\_\_\_

Math 1211: Winter 2014  
Midterm Test #2 — Version B

March 12, 2014

Recitation Section:

- ☐ Section A: 10:00–11:15F (J. Irving)
- ☐ Section B: 2:30–3:45W (J. Irving)
- ☐ Section C: 1:00–2:15F (A. Ellis)
- ☐ Section D: 1:00–2:15F (S. Sikka)
- ☐ Section E: 10:00–11:15F (S. Sikka)

Instructions:

- *No electronic devices or aids of any kind are to be in your immediate possession during the test. Possession of such items will be construed as an act of academic dishonesty.*
- *There are 5 pages plus this cover page. Check that your test paper is complete.*
- *There are a total of 70 marks. The value of each question is indicated in the margin.*
- *Answer in the spaces provided, using backs of pages for additional space if necessary.*
- *Show all your work. Insufficient justification will result in a loss of marks.*

Page	Maximum	Your Score
1	15	
2	13	
3	12	
4	18	
5	12	
Total	70	

1. Evaluate the following integrals.

[5]

(a)  $\int \tan^3 x \sec^3 x \, dx$

[10]

(b)  $\int \frac{x^2}{\sqrt{4-x^2}} \, dx$

**[13]**

(c)  $\int \frac{9x + 1}{(x^2 + 4)(x - 1)^2} dx$

- [12] 2. Determine whether the following improper integrals converge or diverge. If an integral converges, find its value.

(a)  $\int_e^\infty \frac{dx}{x\sqrt{\ln x}}$

(b)  $\int_{-6}^2 \frac{3}{\sqrt[3]{2-t}} dt$

(c)  $\int_0^1 \frac{e^x}{1-x} dx$  [Hint: This is a challenge. Leave it until the end.]

[6]

3. Let  $\mathcal{C}$  be the segment of the curve  $y = \cos x$  between  $x = -\frac{\pi}{2}$  and  $x = \frac{\pi}{2}$ .

(a) Give an expression, in terms of a definite integral, for the **length** of  $\mathcal{C}$ .

(b) Give an expression, in terms of a definite integral, for the **surface area** of the solid obtained by revolving  $\mathcal{C}$  around the  $x$ -axis.

[12]

4. (a) Find the third degree Taylor polynomial  $T_3(x)$  of the function  $f(x) = x^{5/2}$  centred at  $x = 1$ .

(b) Give an estimate of the error  $\left| (1.2)^{5/2} - T_3(1.2) \right|$ . Justify your answer.

[2] 5. (a) Define the  $N$ -th *partial sum* of the series  $\sum_{n=1}^{\infty} a_n$ .

[2] (b) Define precisely what it means to write  $\sum_{n=1}^{\infty} a_n = S$ .

[8] (c) Do the following series converge or diverge? If a series converges, determine its value.

i.  $\sum_{n=1}^{\infty} \frac{n-3}{n+3}$

ii.  $\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$  [Hint:  $\frac{3}{n(n+3)} = \frac{1}{n} - \frac{1}{n+3}$ ]