Math 1211: Recitation #11 **Winter 2014**

Name:	A#:	Section:

1. Determine the radius and interval of convergence of the following power series:

(a) $\sum_{n=0}^{\infty} \frac{3^n}{n!} x^n$

Apply Ratio Test: $\lim_{n\to\infty} \left| \frac{3^{n+1} x^{n+1}}{(n+1)!} \Big/ \frac{3^n x^n}{n!} \right| = \lim_{n\to\infty} \frac{3|x|}{n+1} = 0$

Therefore the series converges for all x.

The radius of convergence is $R = \infty$

The interval of convergence is $(-\infty, \infty)$

(b) $\sum_{n=1}^{\infty} \frac{2^n}{n} (x-1)^n$

We have $\lim_{n\to\infty} \left| \frac{2^{n+1}(x-1)^{n+1}}{n+1} \Big/ \frac{2^n (x-1)^n}{n} \right| = \lim_{n\to\infty} 2|x-1| \left(\frac{n}{n+1}\right)$

$= 2|x-1|.$

By ratio test, the series converges when $2|x-1| < 1$.
That is, when $|x-1| < \frac{1}{2}$, which is the same as $x \in (\frac{1}{2}, \frac{3}{2})$.

So the radius of convergence is $R = \frac{1}{2}$.

Check endpoints: At $x = \frac{3}{2}$, get $\sum_{n=1}^{\infty} \frac{2^n}{n}(x-1)^n = \sum_{n=1}^{\infty} \frac{2^n}{n}\left(\frac{1}{2}\right)^n = \sum_{n=1}^{\infty} \frac{1}{n}$

THIS DIVERGES (harmonic series)

At $x = \frac{1}{2}$ get $\sum_{n=1}^{\infty} \frac{2^n}{n}(x-1)^n = \sum_{n=1}^{\infty} (-1)^n \frac{1}{n}$

THIS CONVERGES (alternating harm. series)

So the interval of convergence is $[\frac{1}{2}, \frac{3}{2})$

2. (a) Use what you know about geometric series to expand $\frac{1}{1+3x}$ as a power series with centre $c = 0$.

We know $\frac{1}{1-x} = 1 + x + x^2 + \cdots$ for $|x| < 1$.

Therefore $\frac{1}{1+3x} = \frac{1}{1-(-3x)}$

$= 1 + (-3x) + (-3x)^2 + (-3x)^3 + \cdots$ for $|-3x| < 1$

$= 1 - 3x + 9x^2 - 27x^3 + \cdots$ for $|x| < \frac{1}{3}$

In Σ-notation: $\frac{1}{1+3x} = \sum_{n=0}^{\infty} (-3)^n x^n$ for $|x| < 1/3$.

(b) What is the interval of convergence of the series in (a)?

The series is valid for $|x| < \frac{1}{3}$.

So the interval of convergence is $(-\frac{1}{3}, \frac{1}{3})$

(c) Integrate the series in (a) to express $\ln(1 + 3x)$ as a power series centred at 0.

From $\frac{1}{1+3x} = 1 - 3x + 9x^2 - 27x^3 + \cdots$

get $\frac{1}{3}\ln(1+3x) = C + x - \frac{3}{2}x^2 + \frac{9}{3}x^3 - \frac{27}{4}x^4 + \frac{81}{5}x^5 - \cdots$

Set $x = 0$ to find $C = 0$.

Thus $\ln(1+3x) = 3x - \frac{9}{2}x^2 + \frac{27}{3}x^3 - \frac{81}{4}x^4 + \frac{243}{5}x^5 - \cdots$

(In Σ-notation, $\ln(1+3x) = \sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n x^n}{n}$)

(d) What is the interval of convergence of the series in (c)?

We know the radius of convergence is the same as before integrating, namely $R = 1/3$. So we check endpoints $x = \pm\frac{1}{3}$.

At $x = 1/3$, get the alternating harmonic series, which CONVERGES.

At $x = -1/3$ get $-\sum \frac{1}{n}$, which DIVERGES.

So the interval of convergence is $(-\frac{1}{3}, \frac{1}{3}]$.