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Math 1211: Recitation #1 **Winter 2012**

Name: SOLUTIONS	A#:	Section:

−1/2 point for each missed constant to max of −1 points total

1. Determine the following antiderivatives.

(a) $\int (6x^3 - 5\sqrt[4]{x} + 4)\, dx$

3

$$= 6 \cdot \frac{1}{4}x^4 - 5 \cdot \frac{4}{5}x^{5/4} + 4x + C$$

$$= \frac{3}{2}x^4 - 4x^{5/4} + 4x + C$$

(b) $\int (x+1)(2x+1)\, dx$

2

$$= \int (2x^2 + 3x + 1)\, dx$$

$$= \frac{2}{3}x^3 + \frac{3}{2}x^2 + x + C$$

(c) $\int \frac{5 - 4x^3 + 2x^6}{x^7}\, dx$

3

$$= \int (5x^{-7} - 4x^{-4} + 2x^{-1})\, dx$$

$$= -\frac{5}{6}x^{-6} + \frac{4}{3}x^{-3} + 2\ln|x| + C$$

(d) $\int \frac{1-z}{\sqrt{z}}\, dz$

2

$$= \int (z^{-1/2} - z^{1/2})\, dz$$

$$= 2z^{1/2} - \frac{2}{3}z^{3/2} + C$$

$$\left[= 2\sqrt{z} - \frac{2}{3}z\sqrt{z} \right]$$

(e) $\int e^z(e^{-z} + 1)\, dz$

2

$$= \int (1 + e^z)\, dz$$

$$= z + e^z + C$$

(f) $\int 2\cos(3t+1)\, dt$

2

$$= \frac{2}{3}\sin(3t+1) + C$$

(g) $\int \frac{dx}{3-4x}\, dx$

2

$$= -\frac{1}{4}\ln|3-4x| + C$$

(h) $\int \frac{3x}{(1+x^2)^3}\, dx$

2

$$= \int 3x(1+x^2)^{-3}\, dx$$

$$= -\frac{3}{4}(1+x^2)^{-2} + C$$

$$\left[= -\frac{3}{4}\frac{1}{(1+x^2)^2} \right]$$

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2. (a) Express the area under the curve $y = \sqrt[3]{x+1}$ between $x = 0$ and $x = 7$ as the limit of a Riemann sum.

$$\lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x \quad , \quad \text{where } \Delta x = \frac{7-0}{n} = \frac{7}{n}$$

$$x_i = 0 + i\Delta x = \frac{7i}{n}$$

2

$$\boxed{= \lim_{n\to\infty} \sum_{i=1}^{n} \frac{7}{n} \sqrt[3]{\frac{7i}{n}+1}}$$

each ✓ = ½ pt

(b) Rewrite your answer to part (a) as a definite integral, and evaluate this integral.

3

$$\int_0^7 \sqrt[3]{x+1}\,dx = \int_0^7 (1+x)^{1/3}\,dx$$

$$= \frac{3}{4}(1+x)^{4/3}\Big|_0^7$$

$$= \frac{3}{4}\left[8^{4/3} - 1^{4/3}\right]$$

$$= \boxed{\frac{45}{4}}$$

3. Evaluate the following definite integrals.

(a) $\displaystyle\int_1^2 \left(\frac{3}{4x^2} - \frac{x^2}{2}\right) dx$

3

$$= \left(-\frac{3}{4}x^{-1} - \frac{1}{6}x^3\right)\Big|_1^2$$

$$= \left(-\frac{3}{8} - \frac{8}{6}\right) - \left(-\frac{3}{4} - \frac{1}{6}\right)$$

$$= -\frac{41}{24} + \frac{22}{24} = \boxed{-\frac{19}{24}}$$

(b) $\displaystyle\int_0^1 \frac{4}{t^2+1}\,dt$

3

$$= 4\tan^{-1} t \Big|_0^1$$

$$= 4\tan^{-1}(1) - 4\tan^{-1}(0)$$

$$= 4 \cdot \frac{\pi}{4} - 4 \cdot 0$$

$$= \boxed{\pi}$$

(c) $\displaystyle\int_{-1/2}^{1} e^{2x+1}\,dx$

2

$$= \frac{1}{2}e^{2x+1}\Big]_{-1/2}^{1}$$

$$= \boxed{\frac{e^3 - 1}{2}}$$

(d) $\displaystyle\int_1^4 \sqrt{z}(1+2z)\,dz$.

3

$$= \int_1^4 \left(z^{1/2} + 2z^{3/2}\right) dz$$

$$= \left(\frac{2}{3}z^{3/2} + \frac{4}{5}z^{5/2}\right)\Big|_1^4$$

$$= \left(\frac{2}{3}\cdot 8 + \frac{4}{5}\cdot 32\right) - \left(\frac{2}{3} + \frac{4}{5}\right)$$

$$= \frac{464}{15} - \frac{22}{15}$$

$$= \boxed{\frac{442}{15}}$$

−½ point for each arithmetic error, to max of −2 points total