

Name: SOLUTIONS

A#:

Section:

$$1. \int x^3(1+2x^4)^2 dx$$

$$\text{Let } u = 1+2x^4 \checkmark$$

$$\Rightarrow du = 8x^3 dx$$

$$= \frac{1}{8} \int u^2 du \checkmark$$

$$= \frac{1}{24} u^3 + C \checkmark \quad (5)$$

$$= \frac{1}{24} (1+2x^4)^3 + C \checkmark$$

$$2. \int \frac{dx}{x \ln x}$$

$$\text{Let } u = \ln x \checkmark$$

$$\Rightarrow du = \frac{1}{x} dx \checkmark$$

$$= \int \frac{du}{u} \checkmark$$

$$= \ln|u| + C \checkmark \quad (4)$$

$$= \ln|\ln x| + C$$

$$3. \int \tan 2x \sec^3 2x dx$$

$$\text{Let } u = \sec 2x \checkmark$$

$$\Rightarrow du = 2 \sec 2x \tan 2x dx$$

$$= \frac{1}{2} \int u^2 du \checkmark$$

$$= \frac{1}{6} u^3 + C \checkmark \quad (4)$$

$$= \frac{1}{6} \sec^3 2x + C \checkmark$$

$$4. \int x^3 \sqrt{x+2} dx$$

$$\text{Let } u = x+2 \checkmark$$

$$du = dx$$

$$\text{and } x = u-2$$

$$= \int (u-2)^3 \sqrt{u} du \checkmark$$

$$= \int (u^{4/3} - 2u^{1/3}) du \checkmark$$

$$= \frac{3}{7} u^{7/3} - \frac{3}{2} u^{4/3} + C \checkmark$$

$$> \frac{3}{7} (x+2)^{7/3} - \frac{3}{2} (x+2)^{4/3} + C$$

(5)

$$5. \int \frac{dz}{1+2\sqrt{z}}$$

$$\text{Let } u = 1+2\sqrt{z} \checkmark$$

$$\Rightarrow du = \frac{1}{\sqrt{z}} dz$$

$$= \int \frac{u-1}{zu} du \checkmark$$

$$= \frac{1}{2} \int \left(1 - \frac{1}{u}\right) du \checkmark$$

$$= \frac{1}{2}(u - \ln|u|) + C \checkmark \quad (5)$$

$$= \frac{1}{2}(1+2\sqrt{z} - \ln(1+2\sqrt{z})) + C$$

[OR simply $\sqrt{z} - \frac{1}{2} \ln(1+2\sqrt{z}) + C$]

$$6. \int \frac{x+4}{x^2+4} dx$$

$$= \int \frac{x}{x^2+4} dx + \int \frac{4}{x^2+4} dx$$

$$= \frac{1}{2} \ln(x^2+4) \checkmark + \int \frac{1}{\frac{x^2}{4}+1} dx \quad u = \frac{x}{2}, du = \frac{1}{2} dx$$

$$= \frac{1}{2} \ln(x^2+4) + 2 \int \frac{du}{u^2+1} \checkmark$$

$$= \frac{1}{2} \ln(x^2+4) + 2 \tan^{-1} u + C \checkmark \quad (6)$$

$$= \frac{1}{2} \ln(x^2+4) + 2 \tan^{-1}\left(\frac{x}{2}\right) + C$$

$$7. \int e^z \cot(e^z) dz$$

$$\text{Let } u = \sin(e^z)$$

$$\Rightarrow du = \cos(e^z) e^z dz$$

$$= \int e^z \frac{\cos e^z}{\sin e^z} dz$$

$$= \int \frac{du}{u} \checkmark$$

$$= \ln|u| + C \checkmark$$

$$= \ln|\sin(e^z)| + C \quad (4)$$

8. Evaluate $\int_0^{1/4} \frac{(x+1) dx}{\sqrt{1-4x^2}}$

$$= \int_0^{1/4} \frac{x}{\sqrt{1-4x^2}} dx + \int_0^{1/4} \frac{1}{\sqrt{1-4x^2}} dx$$

(1) ✓ (2) ✓ Each ✓ is
1/2 point here.

For (1), let $u = 1-4x^2$, so $du = -8x dx$.

$$\text{Get } \int_0^{1/4} \frac{x}{\sqrt{1-4x^2}} dx = -\frac{1}{8} \int_{1}^{3/4} \frac{du}{\sqrt{u}} = -\frac{1}{8} [2\sqrt{u}]_{1}^{3/4} = -\frac{1}{4} \left[\frac{\sqrt{3}}{2} - 1 \right]$$

(5)

For (2), let $u = 2x$ so $du = 2dx$.

$$\text{Get } \int_0^{1/4} \frac{1}{\sqrt{1-4x^2}} dx = \frac{1}{2} \int_0^{1/2} \frac{du}{\sqrt{1-u^2}} = \frac{1}{2} [\sin^{-1} u]_0^{1/2} = \frac{1}{2} \left[\frac{\pi}{6} - 0 \right]$$

Altogether, $\int_0^{1/4} \frac{(x+1) dx}{\sqrt{1-4x^2}} = \boxed{-\frac{\sqrt{3}}{8} + \frac{1}{4} + \frac{\pi}{12}}$

9. Carefully consider the following evaluation of $\int_{-1}^1 \frac{dx}{1+x^4}$.

Let $u = x^2$, so that $du = 2x dx$.

Since $x = \sqrt{u}$ we get $dx = \frac{1}{2\sqrt{u}} du$.

Notice that when $x = -1$ we have $u = 1$, and when $x = 1$ we also have $u = 1$. Therefore

$$\int_{-1}^1 \frac{dx}{1+x^4} = \frac{1}{2} \int_1^1 \frac{du}{\sqrt{u}(1+u^2)}$$

But the integral on the right-hand side must evaluate to zero because we are integrating from 1 to 1.

Therefore

$$\int_{-1}^1 \frac{dx}{1+x^4} = 0.$$

(a) How do you know that the integral *cannot* truly evaluate to 0..

(2) This integral gives the area under the curve $y = \frac{1}{1+x^4}$
between $x = -1$ and $x = 1$. This area must be
positive (ie not zero!) because $\frac{1}{1+x^4}$ is always positive.

(b) What is wrong with the given argument?

The argument fails because for $x \in [-1, 1]$,

(2) writing $u = x^2$ is NOT THE SAME as writing $x = \sqrt{u}$.

(For example, when $x = -1$ we get $u = x^2 = (-1)^2 = 1$.

But $x \neq \sqrt{u}$, because $-1 \neq \sqrt{1}$.)

MORAL: Substitutions must be reversible over the range of the integral.