

Name: SOLUTIONS

A#:

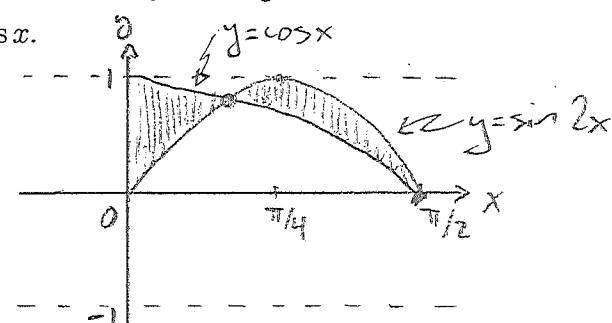
Section:

1. Find the area bounded between $y = \cos x$ and $y = \sin 2x$ on the interval $0 \leq x \leq \frac{\pi}{2}$.
 Begin by sketching this region, being sure to appropriately label your diagram.

Hint: You will require the identity $\sin 2x = 2 \sin x \cos x$.

A quick sketch shows that the curves intersect at two points on the interval $[0, \frac{\pi}{2}]$. To find them, solve $\cos x = \sin 2x$ as follows:

$$\begin{aligned} \cos x &= 2 \sin x \cos x \\ \Rightarrow \cos x - 2 \sin x \cos x &= 0 \\ \Rightarrow \cos x(1 - 2 \sin x) &= 0 \Rightarrow \cos x = 0 \text{ or } \sin x = \frac{1}{2} \\ \Rightarrow x &= \frac{\pi}{2} \text{ or } x = \frac{\pi}{6} \end{aligned}$$

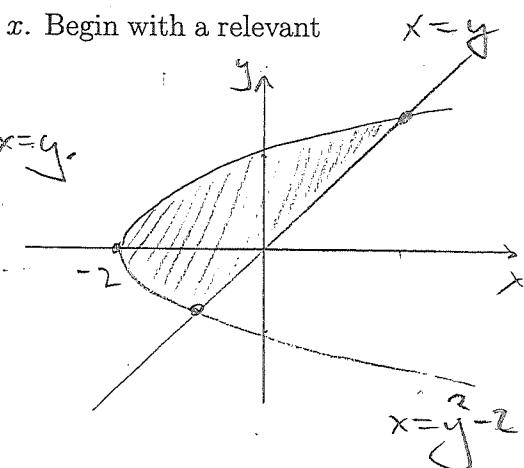


$$\begin{aligned} \text{Thus the desired area is } & \int_0^{\pi/6} (\cos x - \sin 2x) dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x) dx \\ &= \left[\sin x + \frac{1}{2} \cos 2x \right]_0^{\pi/6} + \left[-\frac{1}{2} \cos 2x - \sin x \right]_{\pi/6}^{\pi/2} \\ &= \left(\frac{1}{2} + \frac{1}{2} \cdot \frac{1}{2} \right) - (0 + \frac{1}{2} \cdot 1) + \left(-\frac{1}{2}(-1) - 1 \right) - \left(-\frac{1}{2} \cdot \frac{1}{2} - \frac{1}{2} \right) \\ &= \boxed{\frac{1}{2}} \end{aligned}$$

2. Find the area bounded between the curves $y^2 = x+2$ and $y = x$. Begin with a relevant sketch.

Rewrite curves in terms of y : $x = y^2 - 2$ and $x = y$.

$$\begin{aligned} \text{Find int. pts.: } y^2 &= y + 2 \Rightarrow y^2 - y - 2 = 0 \\ &\Rightarrow (y-2)(y+1) = 0 \\ &\Rightarrow y = -1 \text{ or } y = 2 \end{aligned}$$

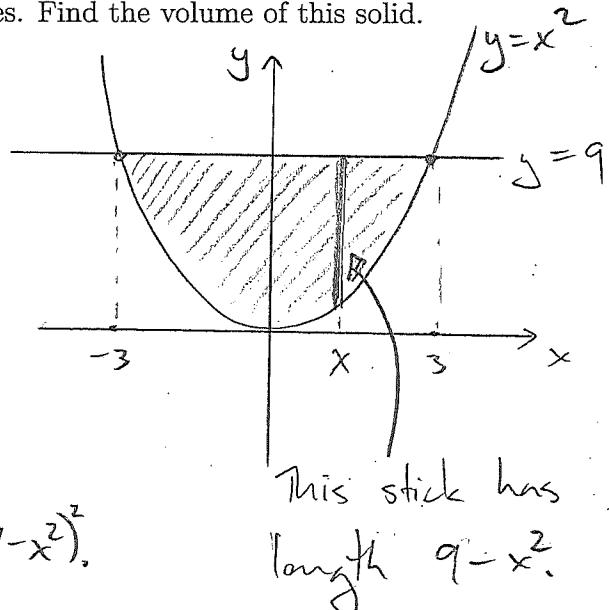


$$\begin{aligned} \text{Thus the desired area is } & \int_{-1}^2 (y - (y^2 - 2)) dy \\ &= \left[\frac{1}{2}y^2 - \frac{1}{3}y^3 + 2y \right]_{-1}^2 \\ &= \boxed{\frac{9}{2}} \end{aligned}$$

3. The base of a solid is the region bounded between the curves $y = x^2$ and $y = 9$, and its cross sections perpendicular to the x -axis are squares. Find the volume of this solid.

The base of the solid is depicted in the diagram.

The cross section through x is a square with side length $9 - x^2$. Its area is therefore $(9 - x^2)^2$.



So the volume is $\int_{-3}^3 (9 - x^2)^2 dx$

$$= \int_{-3}^3 (81 - 18x^2 + x^4) dx$$

$$= [81x - 6x^3 + \frac{1}{5}x^5]_{-3}^3 = \boxed{\frac{1296}{5}}$$

4. Let \mathcal{R} be the region bounded between the curves $y = 1/x^2$, $y = 0$, $x = 1$, and $x = 2$. Sketch the region \mathcal{R} , and find the volume of the solid obtained by revolving \mathcal{R} around the x -axis.

The desired volume is:

$$\pi \int_1^2 \left(\frac{1}{x^2}\right)^2 dx$$

$$= \pi \int_1^2 x^{-4} dx$$

$$= \left[-\frac{\pi}{3} x^{-3} \right]_1^2$$

$$= \boxed{\frac{7\pi}{24}}$$

