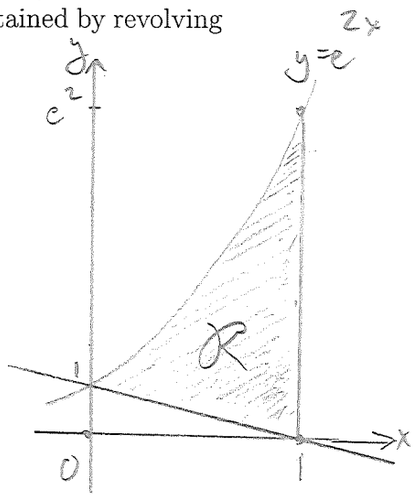
Math 1211: Recitation #4 Winter 2014

Name: SOLUTIONS	A#:	Section:

1. Let $\mathcal{R}$ be the region bounded between the curves $y = e^{2x}$ and $y = 1 - x$, between $x = 0$ and $x = 1$. Sketch the region $\mathcal{R}$, and find the volume of the solid obtained by revolving it around the x-axis.



$$\text{Volume} = \pi \int_0^1 \left((e^{2x})^2 - (1-x)^2 \right) dx$$

$$= \pi \int_0^1 \left(e^{4x} - (1-x)^2 \right) dx$$

$$= \pi \left(\frac{1}{4} e^{4x} + \frac{1}{3}(1-x)^3 \right) \Big|_0^1$$

$$= \pi \left[\frac{1}{4} e^4 + 0 - \frac{1}{4} \cdot 1 - \frac{1}{3} \cdot 1^3 \right]$$

$$= \boxed{\pi \left(\frac{e^4}{4} - \frac{7}{12} \right)}$$

2. Give **expressions** (in terms of definite integrals) for the volumes of the solids obtained by revolving the region $\mathcal{R}$ from Question #1 about the following axes. **Do not evaluate the integrals!**

   - The line $y = -2$.

$$\pi \int_0^1 \left((e^{2x} + 2)^2 - (1 - x + 2)^2 \right) dx$$

   - The line $y = 10$.

$$\pi \int_0^1 \left((10 - (1-x))^2 - (10 - e^{2x})^2 \right) dx$$

   - The y-axis.

By washers: $\pi \int_0^1 \left(1^2 - (1-y^2) \right) dy + \pi \int_1^{e^2} \left(1^2 - \left(\frac{1}{2} \ln y \right)^2 \right) dy$

By shells: $2\pi \int_0^1 x \left(e^{2x} - (1-x) \right) dx$

3. Let $\mathcal{Q}$ be the region bounded between the curves $x = y^2$ and $y = 2x - 1$.

(a) Sketch the region $\mathcal{Q}$. Label all relevant points and curves.

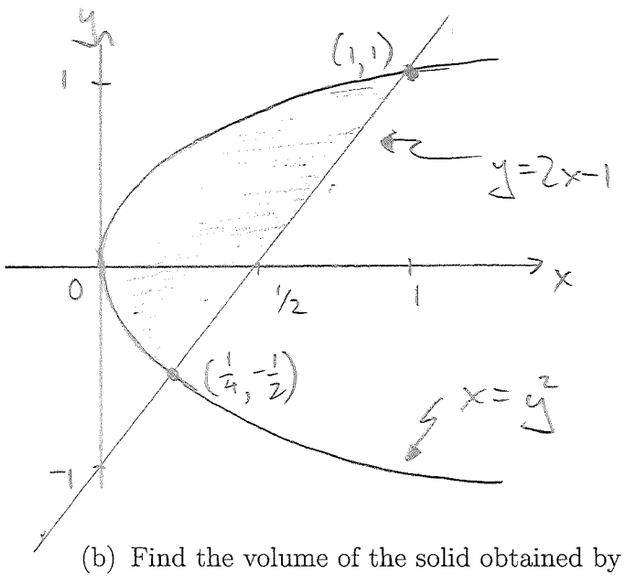
Find the intersection points: $y = 2y^2 - 1$

$\Leftrightarrow 2y^2 - y - 1 = 0$

$\Leftrightarrow (2y+1)(y-1) = 0$

$\Leftrightarrow y = -1/2$ or $y = 1$

$\Rightarrow (x,y) = \left(\frac{1}{4}, -\frac{1}{2}\right)$ or $(1,1)$



(b) Find the volume of the solid obtained by revolving $\mathcal{Q}$ around the y-axis.

Write the curves in terms of y: $x = y^2$ and $x = \frac{y+1}{2}$.

Then the desired volume is

$$\pi \int_{-1/2}^{1} \left(\left(\frac{y+1}{2}\right)^2 - (y^2)^2 \right) dy$$

$$= \frac{\pi}{4} \int_{-1/2}^{1} (y+1)^2 dy - \pi \int_{-1/2}^{1} y^4 dy$$

$$= \frac{\pi}{4} \cdot \frac{1}{3}(y+1)^3 \Big]_{-1/2}^{1} - \pi \cdot \frac{1}{5} y^5 \Big]_{-1/2}^{1}$$

$$= \frac{\pi}{12}\left[8 - \frac{1}{8}\right] - \frac{\pi}{5}\left[1 + \frac{1}{32}\right]$$

$$= \frac{21\pi}{32} - \frac{33\pi}{5 \cdot 32}$$

$$= \frac{72\pi}{5 \cdot 32} = \boxed{\frac{9\pi}{20}}$$