

Name: SOLUTIONS

A#:

Section:

$$1. \int \frac{4x^2 - 5x + 11}{(x^2 - 1)(x^2 + 4)} dx$$

Factor the denominator as $(x-1)(x+1)(x^2+4)$

$$\text{Then set } \frac{4x^2 - 5x + 11}{(x-1)(x+1)(x^2+4)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2+4}$$

$$\Rightarrow A(x+1)(x^2+4) + B(x-1)(x^2+4) + (Cx+D)(x-1)(x+1) = 4x^2 - 5x + 11$$

$$\text{At } x=1 \text{ get } 10A = 10 \Rightarrow A = 1$$

$$\text{At } x=-1 \text{ get } -10B = 20 \Rightarrow B = -2$$

$$\text{At } x=0 \text{ get } 4A - 4B - D = 11 \Rightarrow D = 4A - 4B - 11 = 1$$

Compare coefficients of x^3 to get $A+B+C=0 \Rightarrow C = 1$

$$\begin{aligned} \text{Thus: } \int \frac{4x^2 - 5x + 11}{(x^2 - 1)(x^2 + 4)} dx &= \int \left(\frac{1}{x-1} - \frac{2}{x+1} + \frac{1+x}{x^2+4} \right) dx \\ &= \int \frac{dx}{x-1} - 2 \int \frac{dx}{x+1} + \int \frac{dx}{x^2+4} + \int \frac{x}{x^2+4} dx \\ &= \ln|x-1| - 2 \ln|x+1| + \frac{1}{2} \tan^{-1}\left(\frac{x}{2}\right) + \frac{1}{2} \ln(x^2+4) \\ &\quad + C \end{aligned}$$

2. Find the length of segment of the curve $y = \ln(\cos(x))$ between $x = 0$ and $x = \frac{\pi}{4}$.

$$\begin{aligned}
 \text{length} &= \int_0^{\pi/4} \sqrt{1 + (y')^2} dx \\
 &= \int_0^{\pi/4} \sqrt{1 + \left(\frac{-\sin x}{\cos x}\right)^2} dx \\
 &= \int_0^{\pi/4} \sqrt{1 + \tan^2 x} dx \\
 &= \int_0^{\pi/4} \sec x dx \\
 &= \left. \ln |\sec x + \tan x| \right|_0^{\pi/4} \\
 &= \ln(\sqrt{2} + 1) - \ln(1 + 0) \\
 &= \boxed{\ln(\sqrt{2} + 1)}
 \end{aligned}$$

3. Find the surface area of the solid obtained by revolving the segment of $y = \sqrt{x}$ between $x = 0$ and $x = 2$ about the x -axis.

$$\begin{aligned}
 \text{Area} &= 2\pi \int_0^2 y \sqrt{1 + (y')^2} dx \\
 &= 2\pi \int_0^2 \sqrt{x} \sqrt{1 + \left(\frac{1}{2\sqrt{x}}\right)^2} dx \\
 &= 2\pi \int_0^2 \sqrt{x + \frac{1}{4}} dx \\
 &= 2\pi \cdot \frac{2}{3} \left(x + \frac{1}{4}\right)^{3/2} \Big|_0^2 \\
 &= \frac{4\pi}{3} \left[\left(\frac{9}{4}\right)^{3/2} - \left(\frac{1}{4}\right)^{3/2}\right] \\
 &= \frac{4\pi}{3} \left[\frac{27}{8} - \frac{1}{8}\right] \\
 &= \boxed{\frac{13\pi}{3}}
 \end{aligned}$$