SAINT MARY'S UNIVERSITY
DEPARTMENT OF MATHEMATICS
AND COMPUTING SCIENCE

Name: ______________________________

ID: ______________________________

Math 2321: Linear Algebra II

Midterm Test #1

February 10, 2014

Instructor: J. Irving

Instructions:

- *Calculators are permitted provided they don't dim the lights when you turn them on.*
- *There are 5 pages plus this cover page. Check that your test paper is complete.*
- *There are a total of 80 marks. The value of each question is indicated in the margin.*
- *Show all your work. Insufficient justification will result in a loss of marks.*

Page	**Maximum**	**Your Score**
1	14	
2	26	
3	14	
4	14	
5	12	
Total	**80**	

[14] 1. Give precise definitions for the following terms and notation. Throughout, A is an $n \times n$ matrix.

(a) The *column space* of A

(b) The *nullity* of A.

(c) The *coordinates* of $\mathbf{x} \in \mathbb{R}^n$ with respect to the basis $\mathscr{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$.

(d) An *eigenvector* of A.

(e) The *characteristic polynomial* of A.

(f) The *eigenspace* corresponding to eigenvalue λ of A

(g) The *geometric multiplicity* of the eigenvalue λ of A.

[16] 2. Answer **true** or **false** or **admit ignorance** to the following by circling your choice. You will receive 2 points for a correct response, 1 point for no response, and 0 for an incorrect response. *Throughout, A is an $n \times n$ matrix.*

TRUE	FALSE	UNSURE	The rank and nullity of A sum to n.
TRUE	FALSE	UNSURE	If A has column rank n then $\det A \neq 0$.
TRUE	FALSE	UNSURE	$\det(A^2 - I) = \det(A)^2 - 1$.
TRUE	FALSE	UNSURE	A is invertible if and only if 0 is not an eigenvalue of A.
TRUE	FALSE	UNSURE	Similar matrices have the same eigenvectors.
TRUE	FALSE	UNSURE	The eigenvectors of A are linearly independent.
TRUE	FALSE	UNSURE	If λ is an eigenvalue of A then λ^2 is an eigenvalue of A^2.
TRUE	FALSE	UNSURE	If A is triangular and invertible then A is diagonalizable.

[10] 3. Short answer. **No justification is required**.

(a) If A is a 4×7 matrix, what are the possible values for the nullity of A?

(b) Suppose A and B are 3×3 matrices with $\det A = -1$ and $\det B = 2$. Determine $\det(3A^5B^{-1})$.

(c) Suppose the eigenvalues of a matrix A are $-1, 1$, and 2. Find the eigenvalues of $2A^3 - I$.

(d) Let $A = \begin{bmatrix} 3 & 1 \\ 2 & 2 \end{bmatrix}$. Find the eigenvalues and eigenspaces of A over $\mathbb{Z}_5$, and use this information to find the eigenvalues and eigenspaces of $2A + I$ over $\mathbb{Z}_5$.

[8] 4. Evaluate $\det\begin{bmatrix} 0 & 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 4 & 0 \\ 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & -1 \end{bmatrix}$.

[6] 5. Find all values of k such that the matrix $\begin{bmatrix} k & k+1 & 1 \\ 0 & 2k & k-1 \\ k & 3k & k-2 \end{bmatrix}$ is invertible.

[14] 6. For each of the following matrices A, determine whether A is diagonalizable. If so, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.

(a) $A = \begin{bmatrix} 0 & -1 & -1 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & 1 & 3 \end{bmatrix}$

[8] 7. Let A and B be $n \times n$ matrices with eigenvalues λ and μ, respectively.

(a) Give an example to show that $\lambda\mu$ need not be an eigenvalue of AB.

(b) Suppose λ and μ correspond to the *same* eigenvector $\mathbf{x}$. Show that, in this case, $\lambda\mu$ is an eigenvalue of AB.

[4] 8. Let A and B be invertible matrices of the same dimensions. Show that AB and BA are similar.