Math 2321: Recitation #1

January 10, 2014

1. Let $A = \begin{bmatrix} 1 & 3 & 3 & 2 & -9 \\ -2 & -2 & 2 & -8 & 2 \\ 2 & 3 & 0 & 7 & 1 \\ 3 & 4 & -1 & 11 & -8 \end{bmatrix}$ and determine the following:

   (a) $\text{rank}(A)$

   (b) $\text{nullity}(A)$

   (c) a basis for the row space of A

   (d) a basis for the column space of A

   (e) a basis for the null space of A

2. Repeat Question #1 with A replaced by A^T. (Try to be lazy!)

3. Find a basis for span $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} \right\}$.

4. Let A be a 3×5 matrix.

   (a) Explain why the columns of A must be linearly dependent.

   (b) What are the possible values of $\text{nullity}(A)$?

5. (a) Let A and B be two $n \times n$ matrices. Prove that $\text{rank}(AB) \leq \text{rank}(B)$ and $\text{rank}(AB) \leq \text{rank}(A)$.

   (b) Use part (a) to show that if A is invertible, then $\text{rank}(AB) = \text{rank}(B) = \text{rank}(BA)$.

6. Show that an $n \times m$ matrix A has rank 1 if and only if $A = \mathbf{u}\mathbf{v}^T$, where $\mathbf{u} \in \mathbb{R}^n$ and $\mathbf{v} \in \mathbb{R}^m$.