Math 2321: Recitation #4

February 6, 2014

1. Let A be an $n \times n$ matrix. Give precise and succinct definitions for the following terms and notation.
   (a) The *null space* of A.
   (b) The *column rank* of A.
   (c) The *coordinates* of $\mathbf{x} \in \mathbb{R}^n$ with respect to the basis $\mathscr{B} = \{\mathbf{v}_1, \ldots, \mathbf{v}_n\}$.
   (d) An *eigenvector* of A.
   (e) The *characteristic polynomial* of A.
   (f) The *eigenspace* corresponding to eigenvalue λ of A
   (g) The *geometric multiplicity* of the eigenvalue λ of A.
   (h) The matrix B is *similar* to A if...

2. True or False?
   (a) If A row reduces to R (by elementary row operations), then $\det A = \det R$.
   (b) A is invertible if and only if 0 is an eigenvalue of A.
   (c) Similar matrices have the same eigenvalues.
   (d) If A has n distinct eigenvectors then A is diagonalizable.
   (e) If A is invertible then A is diagonalizable.
   (f) If λ is an eigenvalue of A then $-\lambda$ is an eigenvalue of A^{-1}.
   (g) An $n \times n$ matrix must have at least $\frac{n}{2}$ distinct eigenvalues.

3. Short answer.
   (a) Suppose A and B are 4×4 matrices with $\det A = 2$ and $\det B = 3$. Determine $\det(A^2 B^{-1})$.
   (b) Suppose the eigenvalues of a matrix A are $-1, 1$, and 2. Find the eigenvalues of $3A^2 + 2I$.
   (c) Give an example of a 2×2 matrix that is invertible but not diagonalizable.

4. Evaluate $\det \begin{bmatrix} 2 & 3 & 1 & 2 & 3 \\ 1 & 2 & 1 & 2 & 1 \\ -1 & 1 & 3 & 1 & -1 \\ 1 & 1 & 2 & 1 & 1 \\ 2 & 1 & 0 & -1 & -2 \end{bmatrix}$.

5. Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k.

6. For each of the following matrices A, determine whether A is diagonalizable. If so, find an invertible matrix P and a diagonal matrix D such that $D = P^{-1}AP$.
   (a) $A = \begin{bmatrix} 0 & -2 & 2 \\ 2 & -4 & 3 \\ 2 & -3 & 2 \end{bmatrix}$
   (b) $A = \begin{bmatrix} 2 & 0 & -2 \\ 1 & 3 & 2 \\ 0 & 0 & 3 \end{bmatrix}$