Math 2321: Recitation #9

March 28, 2014

1. (a) Extend the set $\left\{ \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \right\}$ to a basis for $\mathcal{M}_{2\times 2}$.

   (b) Reduce the set $\{1 + x + x^3,\ 2 - x + 2x^3,\ x + x^3,\ x^2 - x,\ 1 - x + x^2,\ x^3\}$ to a basis for $\mathcal{P}_3$.

   (c) Find a basis for span $\left\{1, \sin^2 x, \cos^2 x, \sin 2x, \cos 2x\right\}$ in $\mathcal{F}$.

2. Find a basis for the subspace $\{p(x) : xp'(x) = p(x)\}$ of $\mathcal{P}_3$.

3. Find a formula for the dimension of the vector space of symmetric $n \times n$ matrices over $\mathbb{R}$.

4. Let $\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_n\}$ be a basis of the vector space V. Prove that

$$\{\mathbf{v}_1,\ \mathbf{v}_1 + \mathbf{v}_2,\ \mathbf{v}_1 + \mathbf{v}_2 + \mathbf{v}_3, \ldots, \mathbf{v}_1 + \mathbf{v}_2 + \cdots + \mathbf{v}_n\}$$

   is a also a basis of V.