Math 1210: Recitation Test #1 Winter 2012

Name: SOLUTIONS	A#:

1. Consider the function $f(x) = 5\sqrt{3x} - 2$.

(a) Use the **limit definition** to find the derivative $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left(5\sqrt{3(x+h)} - 2\right) - \left(5\sqrt{3x} - 2\right)}{h}$$

$$= \lim_{h \to 0} 5 \cdot \frac{\sqrt{3(x+h)} - \sqrt{3x}}{h} \cdot \frac{\sqrt{3(x+h)} + \sqrt{3x}}{\sqrt{3(x+h)} + \sqrt{3x}}$$

$$= \lim_{h \to 0} 5 \cdot \frac{3\cancel{h}}{\cancel{h}\left(\sqrt{3(x+h)} - \sqrt{3x}\right)}$$

$$= 5 \cdot \frac{3}{\sqrt{3x} + \sqrt{3x}}$$

$$= \frac{15}{2\sqrt{3x}}$$

(b) Find the equation of the tangent line to the curve $y = f(x)$ at $x = 3$.

From (a), we have $f'(3) = \frac{15}{2\sqrt{9}} = \frac{5}{2}$

Also, $f(3) = 5\sqrt{9} - 2 = 13$

So the tangent line is $\boxed{y - 13 = \frac{5}{2}(x - 3)}$

2. Find $\frac{dy}{dx}$. **Do not** simplify your answers.

(a) $y = 3x^4 - \frac{2}{\sqrt[3]{x}} + 2^x + \frac{7}{5x^2} + 5\tan x + 4e^3 + \frac{x^2+1}{2}$

$$\frac{dy}{dx} = 12x^3 + \frac{2}{3}x^{-4/3} + 2^x \ln 2 - \frac{14}{5x^3} + 5\sec^2 x + 0 + x$$

(b) $y = (2+3x^2)^4 e^{\sqrt{x}+1}$

$$\frac{dy}{dx} = 4(2+3x^2)^3 \cdot 6x \cdot e^{\sqrt{x}+1} + (2+3x^2)^4 e^{\sqrt{x}+1}\left(\frac{1}{2\sqrt{x}}\right)$$

(c) $y = \cos^3(5x) + \sec^3(x^5)$

$$\frac{dy}{dx} = 3\cos^2(5x)(-\sin 5x)\cdot 5 + 3\sec^2(x^5)\cdot \sec(x^5)\tan(x^5)\cdot 5x^4$$

(d) $y = \left(\frac{\sin 2x}{1+\sqrt{1+e^{x^2}}}\right)^6$

$$\frac{dy}{dx} = 6\left(\frac{\sin 2x}{1+\sqrt{1+e^{x^2}}}\right)^5 \cdot \left[\frac{\cos 2x \cdot 2 \cdot (1+\sqrt{1+e^{x^2}}) - \sin 2x \cdot \frac{1}{2}(1+e^{x^2})^{-1/2} \cdot e^{x^2} \cdot 2x}{(1+\sqrt{1+e^{x^2}})^2}\right]$$

3. Point P lies somewhere on a straight line that runs east to west. A particle travels along this line, beginning at time $t = 0$, such that its displacement from a fixed point P at time $t \geq 0$ is given by the formula

$$d(t) = \frac{t^2 + t + 7}{t^2 + 8}.$$

(Positive values of $d(t)$ indicate positions to the *east* of P. Time is measured in seconds and distance in metres.)

(a) Find the average velocity of the particle in the first 2 seconds of travel.

$$v_{avg} = \frac{d(2) - d(0)}{2 - 0} = \frac{\frac{13}{12} - \frac{7}{8}}{2} = \boxed{\frac{5}{48}}$$

(b) What is the instantaneous velocity of the particle at time $t = 2$?

We have $d'(t) = \dfrac{(2t+1)(t^2+8) - 2t(t^2+t+7)}{(t^2+8)^2}$

$$= \frac{-t^2 + 2t + 8}{(t^2+8)^2}$$

So the inst. velocity at $t=2$ is $\boxed{d'(2) = \frac{8}{12^2} = \frac{1}{18} \quad (m/s)}$.

(c) At what time(s) t is the particle stationary (i.e. has velocity 0)?

From (b), get $d'(t) = \dfrac{-(t+2)(t-4)}{(t^2+8)^2}$.

So $d'(t) = 0 \Leftrightarrow t = -2$ or $t = 4$.

Only $t \geq 0$ is allowed, so the particle is stationary at $\boxed{t=4}$

(d) What is the total distance traveled by the particle in the first 10 seconds?

From (c), we know the total distance traveled between $t=0$ and $t=10$ is:

$$|d(4) - d(0)| + |d(10) - d(4)|$$

$$= \left|\frac{9}{8} - \frac{7}{8}\right| + \left|\frac{13}{12} - \frac{9}{8}\right|$$

$$= \frac{1}{4} + \frac{1}{24}$$

$$= \boxed{\frac{7}{24}} \quad \text{(metres)}$$