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# Math 1210: Recitation Test #1 Winter 2012

Name: Solutions A#: ____________

1. Consider the function $f(x) = \dfrac{2}{5x} - 1$.

(a) Use the **limit definition** to find the derivative $f'(x)$.

$$\lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\left(\frac{2}{5(x+h)} - 1\right) - \left(\frac{2}{5x} - 1\right)}{h}$$

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$$= \lim_{h\to 0} \frac{\frac{2}{5(x+h)} - \frac{2}{5x}}{h}$$

$$= 2/5 \lim_{h\to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

$$= 2/5 \lim_{h\to 0} \frac{x - (x+h)}{hx(x+h)}$$

$$= 2/5 \lim_{h\to 0} \frac{-h}{hx(x+h)}$$

$$= 2/5 \lim_{h\to 0} \frac{-1}{x(x+h)}$$

$$= 2/5 \cdot \frac{-1}{x^2}$$

$$\boxed{= \frac{-2}{5x^2}}$$

(b) Find the equation of the tangent line to the curve $y = f(x)$ at $x = 1$.

$$f'(1) = \frac{-2}{5(1)^2} = \frac{-2}{5}$$

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$$f(1) = 2/5 - 1 = -3/5$$

$$y - y_0 = m(x - x_0)$$

$$y + 3/5 = -2/5(x - 1)$$

2. Find $\frac{dy}{dx}$. **Do not** simplify your answers.

(a) $y = 3^x - 2x^5 + \frac{3}{5\sqrt[4]{x}} + \frac{1-x^2}{3} + + \frac{5}{7x^3} + 5\tan x + 4\sqrt{\pi}$

$y' = 3^x \ln 3 - 10x^4 + \frac{3}{20} x^{-5/4} - \frac{2}{3}x \quad - 15/7\, x^{-4} \quad + 5\sec^2 x + C$

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(b) $y = \sin^4(3x) + 2\sec^3(x^4)$

$y' = 4\sin^3(3x) \cdot \cos(3x) \cdot 3 + 2 \cdot 3\sec^2(x^4) \cdot \sec(x^4)\tan(x^4) \cdot 4x^3$

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(c) $y = e^{3\sqrt{x}}(1 + 3x^3)^5$

$y' = e^{3\sqrt{x}} \cdot 5(1+3x^3)^4 \cdot 9x^2 + \frac{3}{2\sqrt{x}} e^{3\sqrt{x}} \cdot (1+3x^3)^5$

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(d) $y = \sqrt[3]{\frac{\cos 2x}{1 + xe^{x^2}}}$

$y' = 1/3 \left( \frac{\cos 2x}{1 + xe^{x^2}} \right)^{-2/3} \cdot \frac{-2\sin 2x\,(1 + xe^{x^2}) - \cos 2x\,(e^{x^2} + 2x^2 e^{x^2})}{(1 + xe^{x^2})^2}$

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3. Point $P$ lies somewhere on a straight line that runs east to west. A particle travels along this line, beginning at time $t = 0$, such that its displacement from a fixed point $P$ at time $t \geq 0$ is given by the formula

$$d(t) = \frac{t^2 - t + 7}{t^2 - 2t + 9}.$$

(Positive values of $d(t)$ indicate positions to the *east* of $P$. Time is measured in seconds and distance in metres.)

(a) Find the average velocity of the particle in the first 2 seconds of travel.

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$$V_{ave} = \frac{d(2) - d(0)}{2 - 0} = \frac{1 - 7/9}{2} = 2/18 = \boxed{1/9 \text{ m/s East}}$$

(b) What is the instantaneous velocity of the particle at time $t = 2$?

$$d'(t) = \frac{(2t-1)(t^2-2t+9) - (2t-2)(t^2-t+7)}{(t^2-2t+9)^2}$$

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$$= \frac{-t^2 + 4t + 5}{(t^2-2t+9)^2}$$

So $$d'(2) = \frac{-(2)^2 + 4(2) + 5}{(2^2 - 2(2) + 9)^2}$$

$$\boxed{= 1/9 \text{ m/s}}$$

(c) At what time(s) $t$ is the particle stationary (i.e. has velocity 0)?

$$0 = -t^2 + 4t + 5$$

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$$= t^2 - 4t - 5$$

$$= (t-5)(t+1)$$

So $\boxed{t = 5}$ or $t = -1$ inadmissible

So the particle is stationary at $t = 5$ s.

(d) What is the total distance traveled by the particle in the first 5 seconds?

$$|d(0) - d(5)|$$

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$$= |7/9 - \frac{27}{24}|$$

$$= |56/72 - 81/72|$$

$$= |-25/72|$$

$$= 25/72 \text{ m}$$