Math 1210: Recitation Test #1 Winter 2012

Name: SOLUTIONS	A#:

1. Consider the function $f(x) = \dfrac{1}{\sqrt{2x}} + 1$.

   (a) Use the **limit definition** to find the derivative $f'(x)$.

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\left(\frac{1}{\sqrt{2(x+h)}} + 1\right) - \left(\frac{1}{\sqrt{2x}} + 1\right)}{h}$$

$$= \lim_{h \to 0} \frac{\sqrt{2x} - \sqrt{2(x+h)}}{h\sqrt{2x}\sqrt{2(x+h)}} \cdot \frac{\sqrt{2x} + \sqrt{2(x+h)}}{\sqrt{2x} + \sqrt{2(x+h)}}$$

$$= \lim_{h \to 0} \frac{-2\cancel{h}}{\cancel{h}\sqrt{2x}\sqrt{2(x+h)}\left(\sqrt{2x} + \sqrt{2(x+h)}\right)}$$

$$= \frac{-2}{\sqrt{2x}\sqrt{2x}(\sqrt{2x} + \sqrt{2x})}$$

$$= \frac{-2}{2x(2\sqrt{2x})}$$

$$= \boxed{-\frac{1}{2x\sqrt{2x}}}$$

   (b) Find the equation of the tangent line to the curve $y = f(x)$ at $x = 2$.

From (a), we have $f'(2) = -\dfrac{1}{4\sqrt{4}} = -\dfrac{1}{8}$

and $f(2) = \dfrac{1}{\sqrt{4}} + 1 = \dfrac{3}{2}$

So the desired eqn is $\boxed{y - \frac{3}{2} = -\frac{1}{8}(x-2)}$

2. Find $\frac{dy}{dx}$. **Do not** simplify your answers.

(a) $y = 3x^7 + 4^x + \tan x + \frac{2}{5\sqrt[4]{x}} + 7e^4 - \frac{2}{3x^5} + \frac{x^2+3}{3}$

$$\frac{dy}{dx} = 21x^6 + 4^x \ln 4 + \sec^2 x - \frac{1}{10}x^{-5/4} + 0 + \frac{10}{3x^6} + \frac{2}{3}x$$

(b) $y = e^{x^2}(1 + 5x^4)^3$

$$\frac{dy}{dx} = e^{x^2} \cdot 2x \cdot (1+5x^4)^3 + e^{x^2} \cdot 3(1+5x^4)^2 \cdot 20x^3$$

(c) $y = \frac{\sin 3x}{1 + x^2 e^{2x}}$

$$\frac{dy}{dx} = \frac{\cos 3x \cdot 3 \cdot (1+x^2e^{2x}) - \sin 3x\left(2xe^{2x} + x^2e^{2x} \cdot 2\right)}{(1+x^2e^{2x})^2}$$

(d) $y = \sqrt{\sec^4(2x) + 3\cos^2(x^4)}$

$$\frac{dy}{dx} = \frac{1}{2}\left[\sec^4 2x + 3\cos^2 x^4\right]^{-1/2} \cdot \Big(4\sec^3(2x) \cdot \sec(2x)\tan(2x) \cdot 2$$
$$+ 6\cos(x^4)(-\sin x^4) \cdot 4x^3\Big)$$

3. Point P lies somewhere on a straight line that runs east to west. A particle travels along this line, beginning at time $t = 0$, such that its displacement from a fixed point P at time $t \geq 0$ is given by the formula

$$d(t) = \frac{t^2 + 5t + 13}{t^2 + 4t + 12}.$$

(Positive values of $d(t)$ indicate positions to the *east* of P. Time is measured in seconds and distance in metres.)

(a) Find the average velocity of the particle in the first 2 seconds of travel.

$$v_{avg} = \frac{d(2) - d(0)}{2 - 0} = \frac{\frac{27}{24} - \frac{13}{12}}{2} = \boxed{\frac{1}{48}}$$

(b) What is the instantaneous velocity of the particle at time $t = 2$?

We have $$d'(t) = \frac{(2t+5)(t^2+4t+12) - (2t+4)(t^2+5t+13)}{(t^2+4t+12)^2}$$

$$= \frac{-t^2 - 2t + 8}{(t^2+4t+12)^2}$$

So the inst. velocity at $t = 2$ is $\boxed{d'(2) = 0}$ (m/s)

(c) At what time(s) t is the particle stationary (i.e. has velocity 0)?

From (b) we have $$d'(t) = \frac{-(t+4)(t-2)}{(t^2+4t+12)^2}$$

So $d'(t) = 0 \Leftrightarrow t = -4$ or $t = 2$

But $t = -4$ is inadmissible, so the particle is stationary only at $t = 2$.

(d) What is the total distance traveled by the particle in the first 6 seconds?

From (c), this will be

$$|d(2) - d(0)| + |d(6) - d(2)|$$

$$= \left|\frac{27}{24} - \frac{13}{12}\right| + \left|\frac{79}{72} - \frac{27}{24}\right|$$

$$= \frac{1}{24} + \frac{1}{36}$$

$$\boxed{= \frac{5}{72}}$$ (metres)