

Name:

A#:

1. Consider the function $f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ \sqrt{x+1} & \text{if } 0 \leq x < 3 \\ 5-x & \text{if } x \geq 3. \end{cases}$

[8] (a) Determine all points x at which $f(x)$ is discontinuous. **Justify** your answer fully.

[3] (b) Decide whether f is continuous from the left or from the right at each of the points of discontinuity found in (a).

[4] (c) Sketch the graph of $y = f(x)$.

- [17] 2. Evaluate the following limits, if they exist. If a limit does not exist, decide whether it tends to $\pm\infty$. **Justify** your answers fully.

(a) $\lim_{x \rightarrow 9} \frac{x - 9}{\sqrt{x} - 3}$

(b) $\lim_{x \rightarrow 2} \frac{|x - 2|}{x^2 - 2x}$

(c) $\lim_{t \rightarrow \pi/2} \frac{1 - \cos t}{1 - \sin t}$

(d) $\lim_{z \rightarrow 1} \frac{e^z}{\sqrt[3]{1 - z}}$

[4] 3. State the Mean Value Theorem as precisely as you can, and draw a picture that illustrates its meaning.

[6] 4. (a) Find all critical numbers of the function $f(x) = \sqrt[3]{x^2 - 2x - 3}$.

[5] (b) Find the absolute maximum and minimum values of $f(x)$ on the interval $[-2, 2]$.

[18] 5. For the the function $g(x) = 2x^3 - 3x^2 - 12x$, do the following:

- Determine where the function is **increasing** and **decreasing**, and find all local **maxima** and **minima**.
- Determine where the function is **concave up** and **concave down**, and find all **inflection points**.
- Find all x - and y - **intercepts**.
- Use this information to sketch the curve $y = g(x)$.