

Name:

A#:

1. Consider the function f defined by the rule $f(x) = \begin{cases} x^2 & \text{if } x \geq 1 \\ x & \text{if } x < 1. \end{cases}$

(a) Sketch the graph of $y = f(x)$.

(b) What is $f'(x)$, for $x < 1$?

(c) What is $f'(x)$, for $x > 1$?

(d) Determine $\lim_{x \rightarrow 1^-} \frac{f(x) - f(1)}{x - 1}$ and $\lim_{x \rightarrow 1^+} \frac{f(x) - f(1)}{x - 1}$.

(e) Use the results of part (d) to explain why $f'(1)$ does not exist.

2. Evaluate the following limits. If a limit does not exist, explain fully why this is the case.

$$(a) \lim_{x \rightarrow 2} \frac{x^2 - x - 2}{x^2 - 4}$$

$$(b) \lim_{x \rightarrow 0} \frac{e^x}{1 - 2 \cos x}$$

$$(c) \lim_{x \rightarrow 4} \frac{5x}{(x - 4)^3}$$

$$(d) \lim_{x \rightarrow 0^+} \cos\left(\frac{1}{x}\right)$$