# Math 1210 – Recitation #2 Winter 2012

| Name: SOLUTIONS | A#: |
|---|---|

1. A particle travels along the straight line shown below, beginning its journey at position $P$ at time $t = 0$. For all $t \geq 0$, its displacement from $P$ at time $t$ is given by the formula $d(t) = t^3 - 12t^2 + 21t$, where positive values of displacement indicate positions to the *east* of $P$, and negative values indicate positions to the *west*. Time is measured in seconds and distance in metres..

West ←———————— $P$ ————————→ East

(a) Where is the particle located after precisely 5 seconds?

$$d(5) = 5^3 - 12(5^2) + 21(5) = 125 - 300 + 105 = -70$$

So the particle is 70m WEST of P after 5 seconds.

(b) Find the average velocity of the particle in the first 10 seconds of travel.

$$v_{AVG} = \frac{d(10) - d(0)}{10 - 0} = \frac{(10^3 - 12 \cdot 10^2 + 21 \cdot 10) - 0}{10} = 1$$

$\therefore$ the avg. velocity is 1 m/s.

(c) Use the limit definition to determine the instantaneous velocity $v(t)$ of the particle at time $t$.

$$v(t) = \lim_{h \to 0} \frac{d(t+h) - d(t)}{h}$$

$$= \lim_{h \to 0} \frac{(t+h)^3 - 12(t+h)^2 + 21(t+h) - (t^3 - 12t^2 + 21t)}{h}$$

$$= \lim_{h \to 0} \frac{t^3 + 3t^2h + 3th^2 + h^3 - 12(t^2 + 2th + h^2) + 21(t+h) - t^3 + 12t^2 - 21t}{h}$$

$$= \lim_{h \to 0} \frac{3t^2h + 3th^2 + h^3 - 24th - 12h^2 + 21h}{h}$$

$$= \lim_{h \to 0} (3t^2 + 3th + h^2 - 24t - 12h + 21)$$

$$= 3t^2 - 24t + 21$$

(d) What is the instantaneous velocity at time $t = 5$?

$$v(5) = 3(5)^2 - 24(5) + 21 = -24$$

So the velocity at $t = 5$ is $-24$ m/s. (ie. 24 m/s WESTWARD)

(e) At what times $t$ is the particle stationary? (i.e. has velocity 0)

We are looking for $t$ such that $v(t)=0$:

$$v(t)=0 \iff 3t^2-24t+21=0$$
$$\iff 3(t-1)(t-7)=0$$
$$\iff t=1 \text{ OR } t=7.$$

So the particle is stationary at times $t=1$ and $t=7$.

(f) At what times $t$ is the particle moving eastward?

The particle is moving eastward when $v(t)>0$.

But $v(t)>0 \iff 3(t-1)(t-7)>0$ [see part (e)]

$$\iff t>7 \text{ OR } t<1.$$

So the particle is moving east when $0 \le t < 1$ and when $t>7$.

Picky point: The question indicates that only $t \geq 0$ should be considered

(g) At what times $t$ is the particle located at point $P$?

The particle is at $P$ when $d(t)=0$.

But $d(t)=0 \iff t^3-12t^2+21t=0$

$$\iff t(t^2-12t+21)=0$$
$$\iff t=0 \text{ OR } t^2-12t+21=0$$
$$\iff t=0 \text{ OR } t=\frac{12 \pm \sqrt{60}}{2}=6\pm\sqrt{15} \quad \left(\text{QUADRATIC FORMULA}\right)$$

All these values of $t$ are valid, since they are non-negative.

So the particle is at $P$ when $t=0$, $t=6-\sqrt{15}$ and $t=6+\sqrt{15}$

(h) What is the total distance traveled in the first 10 seconds? (Be careful: Distance is different than displacement.)

We must simply keep track of when the particle changes direction, which can occur only when its velocity is 0. By (e), this happens at $t=1$ and $t=7$.

Thus the required distance is:

$$|d(1)-d(0)| + |d(7)-d(1)| + |d(10)-d(7)|$$
$$= |10-0| + |-98-10| + |10-(-98)|$$
$$= 10+108+108$$
$$= 226.$$

$\therefore$ total distance travelled is 226m.