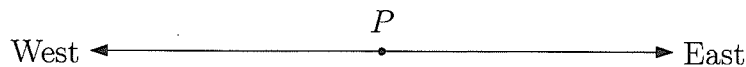


Name: SOLUTIONS

A#:

1. A particle travels along the straight line shown below, beginning its journey at position P at time $t = 0$. For all $t \geq 0$, its displacement from P at time t is given by the formula $d(t) = t^3 - 12t^2 + 21t$, where positive values of displacement indicate positions to the east of P , and negative values indicate positions to the west. Time is measured in seconds and distance in metres.



- (a) Where is the particle located after precisely 5 seconds?

$$\begin{aligned}
 d(5) &= 5^3 - 12(5^2) + 21(5) \\
 &= 125 - 300 + 105 \\
 &= -70
 \end{aligned}
 \left. \vphantom{\begin{aligned} d(5) &= 5^3 - 12(5^2) + 21(5) \\ &= 125 - 300 + 105 \\ &= -70 \end{aligned}} \right\} \begin{array}{l} \text{So the particle is } 70\text{m} \\ \text{WEST of } P \text{ after 5 seconds.} \end{array}$$

- (b) Find the average velocity of the particle in the first 10 seconds of travel.

$$v_{\text{AVG}} = \frac{d(10) - d(0)}{10 - 0} = \frac{(10^3 - 12 \cdot 10^2 + 21 \cdot 10) - 0}{10} = 1$$

\therefore the avg. velocity is 1 m/s.

- (c) Use the limit definition to determine the instantaneous velocity $v(t)$ of the particle at time t .

$$\begin{aligned}
 v(t) &= \lim_{h \rightarrow 0} \frac{d(t+h) - d(t)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{(t+h)^3 - 12(t+h)^2 + 21(t+h) - (t^3 - 12t^2 + 21t)}{h} \\
 &= \lim_{h \rightarrow 0} \frac{t^3 + 3t^2h + 3th^2 + h^3 - 12(t^2 + 2th + h^2) + 21(t+h) - t^3 + 12t^2 - 21t}{h} \\
 &= \lim_{h \rightarrow 0} \frac{3t^2h + 3th^2 + h^3 - 24th - 12h^2 + 21h}{h} \\
 &= \lim_{h \rightarrow 0} (3t^2 + 3th + h^2 - 24t - 12h + 21) \\
 &= 3t^2 - 24t + 21
 \end{aligned}$$

- (d) What is the instantaneous velocity at time $t = 5$?

$$v(5) = 3(5)^2 - 24(5) + 21 = -24$$

$\left. \vphantom{v(5) = 3(5)^2 - 24(5) + 21 = -24} \right\} \begin{array}{l} \text{So the velocity at } t=5 \\ \text{is } -24 \text{ m/s. (ie } 24 \text{ m/s WESTWARD)} \end{array}$

(e) At what times t is the particle stationary? (i.e. has velocity 0)

We are looking for t such that $v(t) = 0$:

$$v(t) = 0 \Leftrightarrow 3t^2 - 24t + 21 = 0$$

$$\Leftrightarrow 3(t-1)(t-7) = 0$$

$$\Leftrightarrow t = 1 \text{ OR } t = 7.$$

So the particle is stationary at times $t = 1$ and $t = 7$.

(f) At what times t is the particle moving eastward?

The particle is moving eastward when $v(t) > 0$.

$$\text{But } v(t) > 0 \Leftrightarrow 3(t-1)(t-7) > 0 \quad [\text{see part (e)}]$$

$$\Leftrightarrow t > 7 \text{ OR } t < 1.$$

So the particle is moving east when $0 \leq t < 1$ and when $t > 7$.

(g) At what times t is the particle located at point P?

The particle is at P when $d(t) = 0$.

$$\text{But } d(t) = 0 \Leftrightarrow t^3 - 12t^2 + 21t = 0$$

$$\Leftrightarrow t(t^2 - 12t + 21) = 0$$

$$\Leftrightarrow t = 0 \text{ OR } t^2 - 12t + 21 = 0$$

$$\Leftrightarrow t = 0 \text{ OR } t = \frac{12 \pm \sqrt{60}}{2} = 6 \pm \sqrt{15} \quad (\text{QUADRATIC FORMULA})$$

All these values of t are valid, since they are non-negative.
So the particle is at P when $t = 0$, $t = 6 - \sqrt{15}$ and $t = 6 + \sqrt{15}$

(h) What is the total distance traveled in the first 10 seconds? (Be careful: Distance is different than displacement.)

We must simply keep track of when the particle changes direction, which can occur only when its velocity is 0. By (e), this happens at $t = 1$ and $t = 7$.

Thus the required distance is:

$$|d(1) - d(0)| + |d(7) - d(1)| + |d(10) - d(7)|$$

$$= |10 - 0| + |-98 - 10| + |10 - (-98)|$$

$$= 10 + 108 + 108$$

$$= 226.$$

\therefore total distance travelled is 226 m.

Picky point: The question indicates that only $t \geq 0$ should be considered