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(Total of 46 marks actually available.)

Math 1210 – Recitation #3 Winter 2012

Name: SOLUTIONS	A#:

1. Find the derivative of each of the following. ***Do not*** **simplify your answers.**

(a) $f(x) = 2x^5 + 7x^{2/3} - \underbrace{\frac{3}{x^4}}_{-3x^{-4}} - 4\pi^3 + \underbrace{\frac{8}{3\sqrt[5]{x}}}_{\frac{8}{3}x^{-1/5}} + \underbrace{2x(1+x^2)}_{2x+2x^3}$

OK if product rule is used

$$f'(x) = 10x^4 + \frac{14}{3}x^{-1/3} + 12x^{-5} + 0 - \frac{8}{15}x^{-6/5} + 2 + 6x$$

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(b) $f(x) = (3x^2+1)^4\left(\frac{2}{x}+1\right)^{3/4}$

$$f'(x) = 4(3x^2+1)^3(6x)\left(\frac{2}{x}+1\right)^{3/4} + (3x^2+1)^4 \cdot \frac{3}{4}\left(\frac{2}{x}+1\right)^{-1/4} \cdot \left(-2x^{-2}\right)$$

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(c) $f(x) = \left(\dfrac{7x + 3\sqrt{x^6+\pi}}{1-x^3}\right)^5$

$$f'(x) = 5\left(\frac{7x+3\sqrt{x^6+\pi}}{1-x^3}\right)^4\left(\frac{\left(7+3\cdot\frac{1}{2}(x^6+\pi)^{-1/2}\cdot 6x^5\right)(1-x^3) - \left(7x+3\sqrt{x^6+\pi}\right)(-3x^2)}{(1-x^3)^2}\right)$$

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2. Find the 10th derivative of $g(x) = \dfrac{1}{1-2x}$.

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$g(x) = (1-2x)^{-1}$

$\Rightarrow g'(x) = -(1-2x)^{-2}(-2) = 2(1-2x)^{-2}$

$\Rightarrow g''(x) = -2\cdot 2(1-2x)^{-3}(-2) = 2\cdot 2^2(1-2x)^{-3}$

$\Rightarrow g'''(x) = -3\cdot 2\cdot 2^2(1-2x)^{-4}(-2) = 3\cdot 2\cdot 2^3(1-2x)^{-4}$

$\Rightarrow g^{(4)}(x) = -4\cdot 3\cdot 2\cdot 2^3(1-2x)^{-5}(-2) = 4\cdot 3\cdot 2\cdot 2^4(1-2x)^{-5}$

The pattern continues, so $\boxed{g^{(10)}(x) = 10\cdot 9\cdot 8\cdot 7\cdots 3\cdot 2\cdot 2^{10}\cdot(1-2x)^{-11}}$

Note: This is more succinctly written as $\dfrac{10!\cdot 2^{10}}{(1-2x)^{11}}$

3. Consider the curve defined by $y = \dfrac{x+1}{x^2+3}$.

(a) What is the equation of the tangent line to this curve at $x = 2$?

We have $y' = \dfrac{1(x^2+3) - 2x(x+1)}{(x^2+3)^2} = \dfrac{3-2x-x^2}{(x^2+3)^2}$ ✓✓✓

So the slope of the desired line is $y'(2) = \dfrac{3-2(2)-2^2}{(2^2+3)^2} = -\dfrac{5}{49}$, ✓

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and it passes through $(2, y(2)) = (2, \frac{2+1}{2^2+3}) = (2, \frac{3}{7})$ ✓

The equation is therefore $\boxed{y - \frac{3}{7} = -\frac{5}{49}(x-2)}$ ✓✓

(b) At what points is the tangent line to the curve horizontal?

We want to find x such that $y'(x) = 0$. ✓

By (a), $y' = \dfrac{-(x-1)(x+3)}{(x^2+3)^2}$ ✓ so $y'(x) = 0 \Leftrightarrow x = 1$ or $x = -3$ ✓ ✓ ✓

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So the tangent line is horizontal at $\boxed{x = 1 \text{ and } x = -3}$

(c) Where does this curve cross the x and y axes?

Note that $y = 0 \Leftrightarrow x+1 = 0 \Leftrightarrow x = -1$, while $y(0) = \frac{0+1}{0^2+3} = \frac{1}{3}$.

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So the x- and y-intercepts are $\boxed{x = -1 \text{ and } y = 1/3}$ ✓ ✓

(d) What happens to the value of y as x becomes very large and positive? What happens to y as x becomes very large and negative?

As x gets large and positive, y becomes small and positive. ✓

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As x gets large and negative, y becomes small and negative. ✓

(e) Use the information obtained in (b)–(d) to sketch this curve.

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