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Math 1210 – Recitation #4 Winter 2012

Name: SOLUTIONS	A#:

1. Sketch the graphs of the curves $y = 1 + \ln(x-2)$ and $y = 1 - e^{1-x}$ on the same set of axes. Label your sketch clearly, including all asymptotes and x, y-intercepts.

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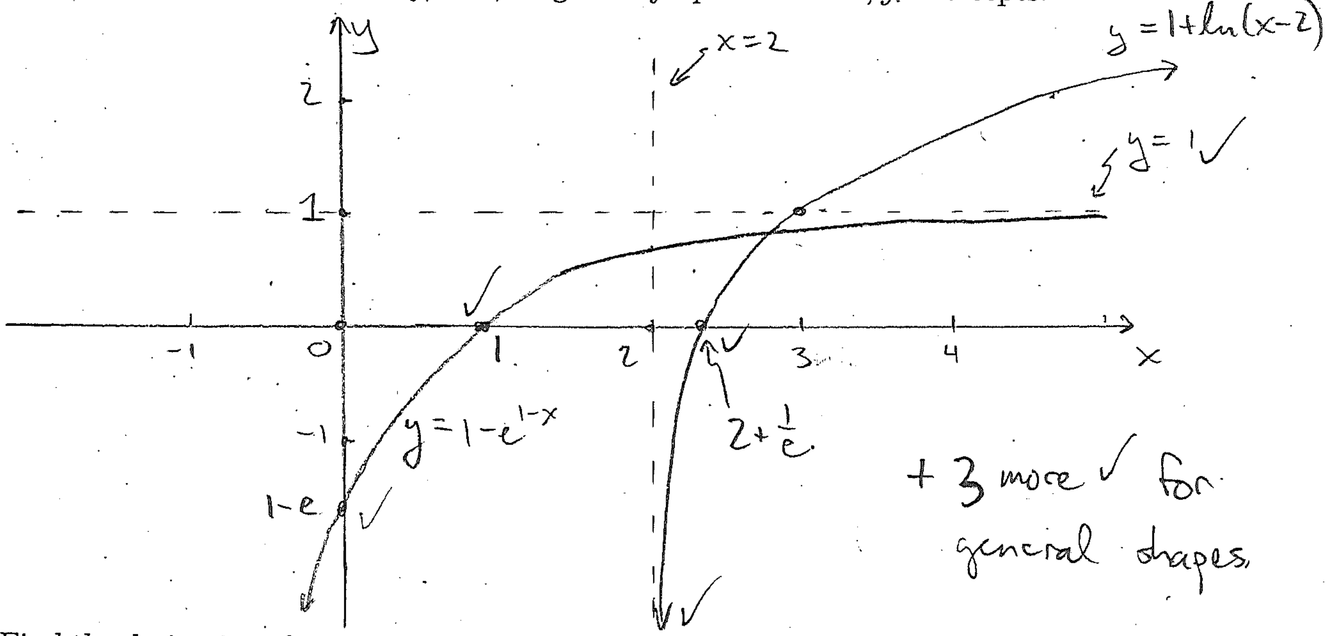

2. Find the derivative of each of the following. You **do not** need to simplify your answers.

(a) $f(x) = x^3 \log_5(x^2 + 5^x)$

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$$f'(x) = 3x^2 \log_5(x^2+5^x) + x^3 \cdot \frac{1}{(x^2+5^x)\ln 5} \cdot (2x + 5^x \ln 5)$$

(b) $f(x) = \ln\sqrt{\dfrac{1+x}{1-2x^2}}$

$$f(x) = \ln\left(\frac{1+x}{1-2x^2}\right)^{1/2} = \frac{1}{2}\ln(1+x) - \frac{1}{2}\ln(1-2x^2)$$

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$$\therefore \boxed{f'(x) = \frac{1}{2} \cdot \frac{1}{1+x} - \frac{1}{2} \cdot \frac{1}{1-2x^2} \cdot (-4x)}$$

3. Use logarithmic differentiation to find $y'(x)$, where

$$y(x) = \frac{e^{x^3}(1+x^2)^x}{(1+e^x)\sqrt[4]{2+\sin x}}$$

$$\ln y = x^3 + x\ln(1+x^2) - \ln(1+e^x) - \frac{1}{4}\ln(2+\sin x)$$

$$\therefore \frac{1}{y}\cdot y' = 3x^2 + \ln(1+x^2) + x\cdot\frac{1}{1+x^2}\cdot 2x - \frac{1}{1+e^x}\cdot e^x - \frac{1}{4}\cdot\frac{1}{2+\sin x}\cdot\cos x$$

$$\therefore y' = \frac{e^{x^3}(1+x^2)^x}{(1+e^x)\sqrt[4]{2+\sin x}}\left[3x^2 + \ln(1+x^2) + \frac{2x^2}{1+x^2} - \frac{e^x}{1+e^x} - \frac{\cos x}{4(2+\sin x)}\right]$$

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4. Consider the function $f(x) = \ln(x^3 - 1) + 2$, defined for $x > 1$.

(a) Find an explicit formula for the inverse function $f^{-1}(x)$.

Let $y = \ln(x^3-1)+2$. Then $\ln(x^3-1) = y-2$

$\Rightarrow x^3 - 1 = e^{y-2}$

$\Rightarrow x = \sqrt[3]{1+e^{y-2}}$

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So $\boxed{f^{-1}(x) = \sqrt[3]{1+e^{x-2}}}$

(b) What are the domain and range of f^{-1}?

From (a), we see that the domain of f^{-1} is all of $\mathbb{R}$.

The range of f^{-1} is the domain of f, which is $(1,\infty)$.

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