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Math 1210 – Recitation #5 **Winter 2012**

Name: SOLUTIONS	A#:

1. Consider the function f defined by the rule $f(x) = \begin{cases} x^2 & \text{if } x \geq 1 \\ x & \text{if } x < 1. \end{cases}$

(a) Sketch the graph of $y = f(x)$.

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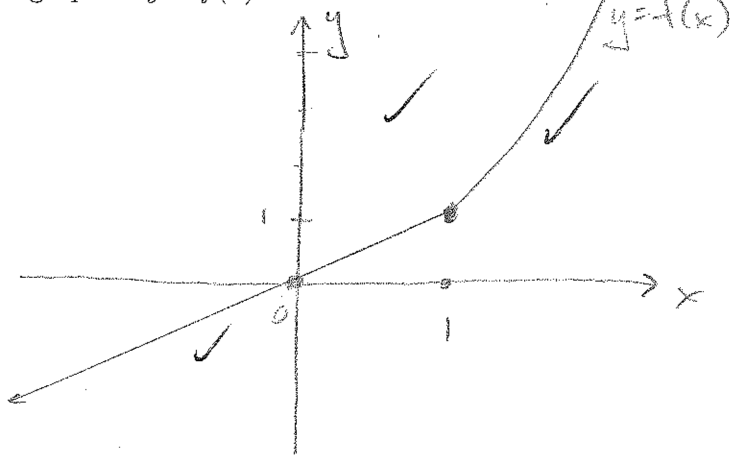

(b) What is $f'(x)$, for $x < 1$?

/2 Since $f(x) = x$ for $x<1$, get $f'(x) = 1$ for $x<1$

(c) What is $f'(x)$, for $x > 1$?

/2 Since $f(x) = x^2$ for $x>1$, get $f'(x) = 2x$ for $x>1$

(d) Determine $\lim_{x\to 1^-} \dfrac{f(x) - f(1)}{x - 1}$ and $\lim_{x\to 1^+} \dfrac{f(x) - f(1)}{x - 1}$.

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$= \lim_{x\to 1^-} \dfrac{x-1}{x-1}$

$= \lim_{x\to 1^-} 1$

$\boxed{= 1}$

$= \lim_{x\to 1^+} \dfrac{x^2-1}{x-1}$

$= \lim_{x\to 1^+} (x+1)$

$\boxed{= 2}$

(e) Use the results of part (d) to explain why $f'(1)$ does not exist.

We have $f'(1) = \lim_{x\to 1} \dfrac{f(x)-f(1)}{x-1}$.

But this limit cannot exist, because part (d) shows the associated right and left limits are unequal.

2. Evaluate the following limits. If a limit does not exist, explain fully why this is the case.

(a) $\lim_{x\to 2} \frac{x^2 - x - 2}{x^2 - 4}$

$= \lim_{x\to 2} \frac{(x-2)(x+1)}{(x-2)(x+2)}$ ✓✓

$= \lim_{x\to 2} \frac{x+1}{x+2}$ ✓

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$\boxed{= \frac{3}{4}}$ ✓

(b) $\lim_{x\to 0} \frac{e^x}{1 - 2\cos x}$

$= \frac{e^0}{1 - 2\cos 0}$

Direct substitution works here because the denominator does not tend to 0 as $x \to 0$

✓

$= \frac{1}{-1}$

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$\boxed{= -1}$ ✓

(c) $\lim_{x\to 4} \frac{5x}{(x-4)^3}$

✓

This limit $\overbrace{\text{does not exist}}$ because, for instance

$$\lim_{x\to 4^+} \frac{5x}{(x-4)^3} = +\infty.$$

/3 ✓✓

To see this, note that $5x \to 20^+$ as $x \to 4^+$, while $(x-4)^3 \to 0^+$ as $x \to 4^+$.

(d) $\lim_{x\to 0^+} \cos\left(\frac{1}{x}\right)$

✓ ✓

This does not exist, since $\overbrace{\frac{1}{x} \to +\infty \text{ as } x \to 0^+}$

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and the cosine function simply oscillates between -1 and 1 as its argument gets large. ✓