Math 1210 – Recitation #6 /36 Winter 2012

Name: SOLUTIONS	A#:

1. The function g is defined by

$$g(x) = \begin{cases} \dfrac{x^2+x-2}{|x-1|} & \text{if } x \neq 1 \\ 2 & \text{if } x = 1. \end{cases}$$

Determine whether g is continuous at $x = 1$, and sketch the graph of $y = g(x)$

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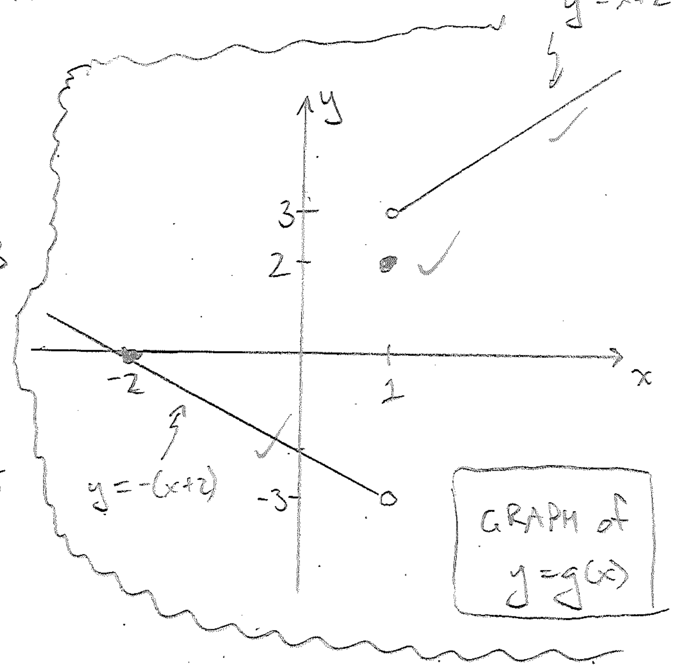
Note that: $\dfrac{x^2+x-2}{|x-1|} = \dfrac{(x-1)(x+2)}{|x-1|} = \begin{cases} x+2, & \text{for } x>1 \\ -(x+2), & \text{for } x<1 \end{cases}$

Thus: $\lim\limits_{x\to 1^+} g(x) = \lim\limits_{x\to 1^+}(x+2) = 3$

$\lim\limits_{x\to 1^-} g(x) = \lim\limits_{x\to 1^-} -(x+2) = -3$

Since these limits are not equal, $\lim\limits_{x\to 1} g(x)$ does not exist. Thus g is discontinuous at $x=1$.



2. Consider the function

$$f(x) = \begin{cases} x+1 & \text{if } x \leq 0 \\ x^2 - x & \text{if } 0 < x < 2 \\ \sqrt{x+2} & \text{if } x \geq 2 \end{cases}$$

Determine all numbers at which f is discontinuous. At which of these numbers is the function continuous from the right, from the left, or neither? Sketch the graph of f.

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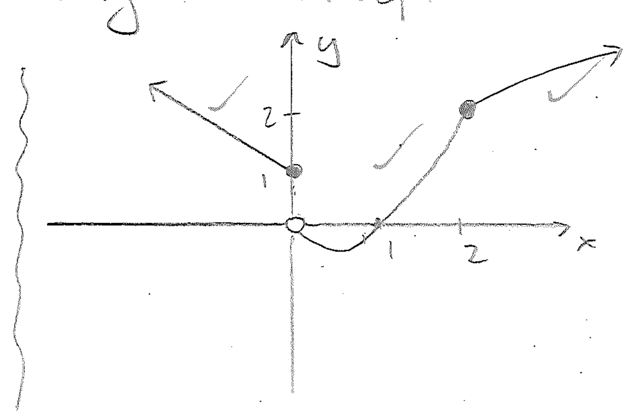
We know that $f(x)$ is continuous everywhere except possibly at $x=0$ and $x=2$.

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} \sqrt{x+2} = 2$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} (x^2-x) = 2$

$\Rightarrow \lim\limits_{x\to 2} f(x) = 2$

Since $f(2) = \sqrt{2+2} = 2$, we conclude that f is continuous at 2.



$\lim\limits_{x\to 0^+} f(x) = \lim\limits_{x\to 0^+} (x^2-x) = 0$

$\lim\limits_{x\to 0^-} f(x) = \lim\limits_{x\to 0^-} (x+1) = 1$

$\Rightarrow \lim\limits_{x\to 0} f(x)$ does not exist, so f is $\boxed{\text{discontinuous at } 0}$

But $f(0) = 0+1 = 1$, so f is $\boxed{\text{continuous from the LEFT at } 0}$

3. Find all critical values of the function $f(x) = (2x - x^2)^{2/3}$.

Get $f'(x) = \frac{2}{3}(2x - x^2)^{-2/3}(2-2x)$

$= \dfrac{4(1-x)}{(x(2-x))^{2/3}}$ ✓✓

So $f'(x) = 0 \iff 1 - x = 0 \iff x = 1$ ✓

and $f'(x)$ is undefined $\iff x(2-x) = 0$ ✓

$\iff x = 0$ OR $x = 2$ ✓

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$\therefore$ the critical values of f are $\boxed{x = 0, 1, 2}$ ✓

4. Find the absolute maximum and minimum values of the function $h(x) = \dfrac{4x+3}{x^2+1}$ over the interval $[-3, 3]$.

Compute: $h'(x) = \dfrac{4(x^2+1) - 2x(4x+3)}{(x^2+1)^2}$

$= \dfrac{-4x^2 - 6x + 4}{(x^2+1)^2}$

$= -\dfrac{2(x+2)(2x-1)}{(x^2+1)^2}$ ✓✓✓

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Then $f'(x)$ is defined for all x, and

$f'(x) = 0 \iff -2(x+2)(2x-1) = 0$ ✓

$\iff x = -2$ OR $x = 1/2$.

Compute: $\left.\begin{array}{l} f(-3) = -9/10 \\ f(-2) = -1 \\ f(1/2) = 4 \\ f(3) = 3/2 \end{array}\right\}$ ✓✓

So: $\boxed{\begin{array}{l} \text{Absolute MAX is } 4 \text{ (at } x = \tfrac{1}{2}) \\ \text{Absolute MIN is } -1 \text{ (at } x = -2) \end{array}}$ ✓ ✓