Math 1210 – Recitation #7 Winter 2012

Name: SOLUTIONS	A#:

1. Evaluate the following limits, if they exist. **Provide proper reasoning** for your answers.

(a) $\lim_{x\to\infty} \frac{3x^3 - x}{2x^3 - 5x^2 + 1}$

$= \lim_{x\to\infty} \frac{3 - 1/x^2}{2 - 5/x + 1/x^2}$ (divide top & bottom by x^3)

$= \boxed{\frac{3}{2}}$

(b) $\lim_{x\to\pi} \frac{\tan 3x}{\sin 2x}$ $\left(\frac{0}{0} \text{ I.F.}\right)$

$\overset{H}{=} \lim_{x\to\pi} \frac{\sec^2(3x)\cdot 3}{\cos(2x)\cdot 2}$

$= \frac{(-1)^2\cdot 3}{1\cdot 2}$

$= \boxed{\frac{3}{2}}$

(c) $\lim_{t\to 0} \frac{t^2 e^t}{2\cos t - t^2 - 2}$ $\left(\frac{0}{0} \text{ I.F}\right)$

$\overset{H}{=} \lim_{t\to 0} \frac{2te^t + t^2e^t}{-2\sin t - 2t}$ $\left(\frac{0}{0} \text{ I.F}\right)$

$\overset{H}{=} \lim_{t\to 0} \frac{2e^t + 2te^t + 2te^t + t^2e^t}{-2\cos t - 2} = \frac{2+0+0+0}{-2-2} = \boxed{-\frac{1}{2}}$

(d) $\lim_{x\to 0^-} e^{1/x}\ln x^2$

$= \lim_{x\to 0^-} \frac{2\ln x}{e^{-1/x}}$ $\left(\frac{-\infty}{\infty} \text{ I.F.}\right)$

$\overset{H}{=} \lim_{x\to 0^-} \frac{2\cdot\frac{1}{x}}{e^{-1/x}\cdot\left(\frac{1}{x^2}\right)}$

$= \lim_{x\to 0^-} 2xe^{1/x} = 0$ (since $e^{1/x} \to 0$ as $x \to 0^-$)

2. Consider the curve $y = \dfrac{\sqrt{2x^2+1}}{x-1}$

(a) Find all vertical asymptotes. (Be sure to compute limits from the left and right of each.)

Possible V.A. at $x=1$. Check that

$$\lim_{x\to 1^-} \frac{\sqrt{2x^2+1}}{x-1} = -\infty \quad \text{and} \quad \lim_{x\to 1^+} \frac{\sqrt{2x^2+1}}{x-1} = +\infty.$$

So $\boxed{x=1}$ is a V.A.

(b) Find all horizontal asymptotes. (Be sure to compute limits to both $+\infty$ and $-\infty$.)

Since $x=\sqrt{x^2}$ for $x>0$, get $\displaystyle\lim_{x\to\infty} \frac{\sqrt{2x^2+1}}{x} = \sqrt{2+1/x^2} = \sqrt{2}$

Since $x=-\sqrt{x^2}$ for $x<0$, get $\displaystyle\lim_{x\to -\infty} \frac{\sqrt{2x^2+1}}{x} = -\sqrt{2+1/x^2} = -\sqrt{2}$

Thus $y=\sqrt{2}$ and $y=-\sqrt{2}$ are H.A.'s.

(c) Find all local maxima and minima.

Compute: $$y'(x) = \frac{\frac{1}{2}(2x^2+1)^{-1/2}(4x)(x-1) - \sqrt{2x^2+1}}{(x-1)^2}$$

$$= \frac{2x(x-1)-(2x^2+1)}{(x-1)^2\sqrt{2x^2+1}} = \frac{-(2x+1)}{(x-1)^2\sqrt{2x^2+1}}$$

So $x=-\frac{1}{2}$ is the only critical number. Note that $y'(x)>0$ for $x<-1/2$ and $y'(x)<0$ for $x>-1/2$.

So there is a local max at $x=-1/2$ with value $y(-\frac{1}{2}) = -\sqrt{\frac{2}{3}}$.

(d) Use (a)–(c) to sketch the curve. Label all asymptotes and local maxima/minima.