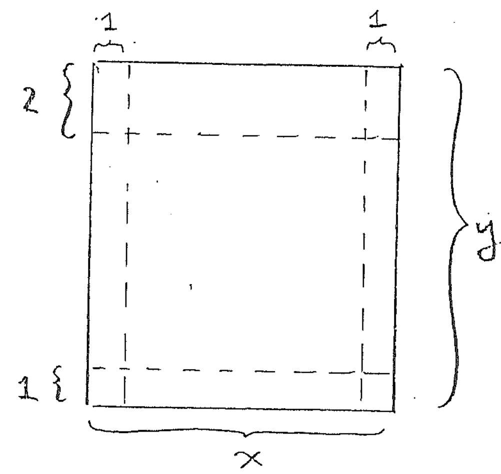
Name: SOLUTIONS	A#:

1. A poster is to have an area of 180 square inches, with 1-inch margins at the bottom and sides and a 2-inch margin at the top. What dimensions will give the largest printed area?



Let x and y be as indicated in the diagram.

Then we wish to maximize

$$A = (x-2)(y-3)$$

subject to the conditions $x \geq 2$, $y \geq 3$ and $xy = 180$

Now: $A = xy - 3x - 2y + 6$

Since $xy = 180 \Rightarrow y = 180/x$, we get

$$A(x) = 180 - 3x - 2\left(\frac{180}{x}\right) + 6$$

$$= 186 - 3x - \frac{360}{x}.$$

Moreover, the conditions $xy = 180$ and $y \geq 3$ force $x \leq 60$.

Thus we want to maximize $A(x)$ on the interval $[2, 60]$.

We have $A'(x) = -3 + \frac{360}{x^2}$

Therefore $A'(x) = 0 \Leftrightarrow 360/x^2 = 3 \Leftrightarrow x^2 = 120 \Rightarrow x = 2\sqrt{30}$.

Now $A(2) = 0$, $A(60) = 0$, and $A(2\sqrt{30}) = 186 - 6\sqrt{30} - \frac{360}{2\sqrt{30}}$

$$= 186 - 12\sqrt{30}.$$

So the maximum printed area occurs when the page is $2\sqrt{30}$ inches wide and $\frac{180}{2\sqrt{30}} = 3\sqrt{30}$ inches tall.

2. Find the point on the line $y = 9 - 6x$ that is closest to the point $(-3, 1)$.

The distance between (x, y) and $(-3, 1)$ is

$$\sqrt{(y-1)^2 + (x+3)^2}$$

Thus we wish to find $x \in \mathbb{R}$ such that

$$\sqrt{(9-6x-1)^2 + (x+3)^2} \quad ✪$$

is minimal. To do this, let

$$d(x) = (8-6x)^2 + (x+3)^2,$$

so that ✪ is minimized when $d(x)$ is minimized.

Get: $d'(x) = 2(8-6x)(-6) + 2(x+3)$
$= 74x - 90$

and $d''(x) = 74$.

Thus $d'(x) = 0 \iff x = \frac{90}{74} = \frac{45}{37}$, and this is a local minimum since $d''\left(\frac{45}{37}\right) > 0$. Moreover, $\lim_{x \to \pm\infty} d(x) = \infty$, so $x = 45/37$ is a global minimum.

Thus the desired point is $(x, y) = \left(\frac{45}{37}, 9 - 6 \cdot \frac{45}{37}\right) = \left(\frac{45}{37}, \frac{63}{37}\right)$.

3. Give an expression, in terms of a limit, for the area under the curve $y = \sqrt{2+x}$ between $x = 2$ and $x = 7$. Express A as a limit.

$$\text{Area} = \lim_{n \to \infty} \sum_{i=1}^{\infty} \sqrt{2 + \left(2 + i \cdot \frac{5}{n}\right)} \cdot \frac{5}{n}$$

$$= \lim_{n \to \infty} \frac{5}{n} \sum_{i=1}^{n} \sqrt{4 + \frac{5i}{n}}$$

MANY EQUIVALENT ANSWERS ARE POSSIBLE.

Note:

$$A = \lim_{n \to \infty} \sum_{i=1}^{\infty} \sqrt{2 + x_i}\, \Delta x$$

where:

$$\Delta x = \frac{7-2}{n} = \frac{5}{n}$$

$$x_i = 2 + i\Delta x = 2 + \frac{5i}{n}$$