Math 1211: Rough Guide to Chapter 11 Fall 2011

Section 11.2

Highlights:

- The use of sigma notation ($\sum$) as shorthand for series.
- The definition of *partial sums* of a given series.
- The definition of an infinite sum as a limit of partial sums.
- Convergence versus divergence of infinite series.
- Analysis of special series: geometric, harmonic, and telescoping.
- The meaning and application of the "*Test for Divergence*" (Theorem 7 in the text).

Notes:

- After mastering this section you should be able to find the sum of any geometric series and a few special telescoping series. But that is it! Unless a series is of one of these two types, you simply won't be able to find its sum (even if it converges!).

  The main point is to understand the *meaning* of infinite series. Evaluating them (i.e. finding their numerical value) is outrageously difficult in practice!.

- I didn't write down the "Test for Divergence" as formally in class as you will find it in your text, but we did discuss this idea at some length.

  The key is to understand the logic behind a very simple observation: If we want to achieve a finite result upon summing infinitely many numbers, then those numbers *must eventually become very small*!

  Therefore, if we notice that the terms in a series *do not approach zero*, then we can safely conclude that the series diverges. (This is the content of Theorem 7.)

  The big **BUT** attached to the previous point is that its converse is *false*. That is, having the terms of a series tend to zero *does not guarantee* that the series converges! For example, the terms of the harmonic series diminish to zero, yet the series itself diverges.

  In summary: If the terms of a series don't tend to zero then the series *diverges*. But if the terms do tend to zero, then we can draw no conclusions!

Practice problems: #2, 9(b), 11–27 (including evens), 29, 30, 31, 35, 37

Section 11.8

Highlights:

- The definition of power series.
- Using the ratio test to determine the radius of convergence of a power series.

Notes:

- I did not discuss power series of the form $\sum_{n=1}^{\infty} c_n(x-a)^n$. These are defined on page 723 of the text, and you should find that they are a very minor generalization of the series we covered in class. Problems 15–25 involve these more general series.
- The problems in this section ask for both the *radius of convergence* and the *interval of convergence* of various power series. We did not have time to discuss the interval of convergence in detail, so you may safely ignore this part of the problems.

  If you are checking your solutions against those in the back of the text, you will likely notice that the solutions in the text are much longer than yours. This is because they are performing some tests to determine the interval of convergence. You can ignore everything in the solutions after the radius of convergence has been established.

Practice problems: #1, 2(a), 3–23 (including evens)

Section 11.9

Highlights:

- Representing functions as power series.
- Manipulating power series via differentiation, integration, and substitution.

Notes:

- We covered this material quite informally in lecture, and I don't expect you to know it at a deep level.
- The point here is to explore manipulations of the simple formula

  $$1 + x + x^2 + x^3 + \cdots = \frac{1}{1-x}$$

  for the sum of a geometric series. We begin by viewing this formula as a *power series representation* of the function $\frac{1}{1-x}$, valid for $|x| < 1$. Now that we're regarding both sides of the formula as functions, we try a variety of functional manipulations, such as differentiation, integration, and substitution.
- You will likely find that you learn this material best by example. Carefully read through the examples done in class and in the text before trying the problems.

Practice problems: #3–9, 13–18, 23–25 (evens included throughout)

Section 11.10

Highlights:

- Finding a power series representation of a given function.
- Maclaurin series.

Notes:

- We only discussed the *Maclaurin series* of a function $f(x)$, given by Formula 7 on page 736 of the text.

  I *do not* expect you to know the more general expressions given in Theorem 5 and Formula 6, nor are you expected to know any of the content of Theorems 8 and 9.

- Let the problems be your guide as to what you are expected to know. You should be able to compute the Maclaurin series of various functions, as in Examples 1, 4, 5, 6, and 8 of the text. (Ignore the final part of Example 4 at the top of page 740.) You should also be able to perform computations such as Example 10(a) and 11.
- You are not responsible for Theorem 17. (I originally assigned Problems 25–28, but I have removed these in the problem list below.)

Practice problems: #5–10, 29–34 (including evens)