MAT1211

Name SOLUTIONS

Quiz # 11A

ID ______

1. Find the area bounded by the curve $r = 2\cos\theta$.

This is a circle with centre $(1,0)$ and radius 1, so its area is π.

$\boxed{5}$ OR $\text{Area} = 2\cdot\frac{1}{2}\int_0^{\pi/2} r^2\,d\theta = \int_0^{\pi/2} 4\cos^2\theta\,d\theta = 2\int_0^{\pi/2}(1+\cos 2\theta)\,d\theta = \pi$

2. Find the entire length of the curve $r = 1 + \cos\theta$.

$$\text{length} = 2\int_0^{\pi}\sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\,d\theta \quad \left[\begin{array}{c}\text{SEE}\\ \text{PICTURE}\end{array}\right]$$

$\boxed{5}$

$$= 2\int_0^{\pi}\sqrt{\sin^2\theta + 1 + 2\cos\theta + \cos^2\theta}\,d\theta$$

$$= 2\sqrt{2}\int_0^{\pi}\sqrt{1+\cos\theta}\,d\theta$$

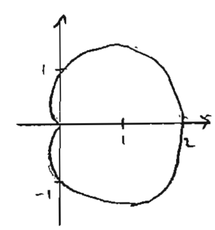
LET $x = \cos\theta \Rightarrow \theta = \cos^{-1}x \Rightarrow d\theta = \dfrac{-dx}{\sqrt{1-x^2}}$

$$= -2\sqrt{2}\int_1^{-1}\frac{\sqrt{1+x}}{\sqrt{1-x^2}}\,dx$$

$$= -2\sqrt{2}\int_1^{-1}\frac{dx}{\sqrt{1-x}} \quad \left(\begin{array}{c}\text{IMPROPER}\\ @\,x=1\end{array}\right)$$

$$= -2\sqrt{2}\cdot\lim_{t\to 1^-}\left[-2\sqrt{1-x}\right]_{x=t}^{x=-1}$$

$$= 8$$



3. Determine whether the sequence $\left\{\dfrac{4n-1}{3n+2}\right\}$ converges or diverges and if it converges find its limit.

$\boxed{2}$ We have $\lim\limits_{n\to\infty}\dfrac{4n-1}{3n+2} = \lim\limits_{n\to\infty}\dfrac{4-\frac{1}{n}}{3+\frac{2}{n}} = \dfrac{4}{3}$, so $\left\{\dfrac{4n-1}{3n+2}\right\}$ converges to $\dfrac{4}{3}$.

4. Determine whether the sequence $\left\{\dfrac{1}{5^n}\right\}$ is increasing or decreasing or not monotonic.

$\boxed{2}$ Since $5^{n+1} = 5\cdot 5^n > 5^n$, we have $\dfrac{1}{5^{n+1}} < \dfrac{1}{5^n}$. Thus $\left\{\dfrac{1}{5^n}\right\}$ is decreasing.

5. Determine with reasons whether the following series are convergent or divergent:

(a) $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$ (if convergent, find its value) NOTE: $\frac{1}{(2n-1)(2n+1)} = \frac{1}{2}\left[\frac{1}{2n-1} - \frac{1}{2n+1}\right]$

$\boxed{2}$ The n^{th} partial sum is $S_n = \frac{1}{2}\left[1-\cancel{\frac{1}{3}}\right] + \frac{1}{2}\left[\cancel{\frac{1}{3}}-\cancel{\frac{1}{5}}\right] + \frac{1}{2}\left[\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right] + \cdots + \frac{1}{2}\left[\cancel{\frac{1}{2n-1}} - \frac{1}{2n+1}\right]$

$= \frac{1}{2}\left[1 - \frac{1}{2n+1}\right]$. Since $S_n \to \frac{1}{2}$, $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)} = \frac{1}{2}$. (CONVERGES)

(b) $\sum_{n=1}^{\infty} \frac{1}{3+e^{-n}}$

$\boxed{2}$ Since $\lim_{n\to\infty} \frac{1}{3+e^{-n}} = \frac{1}{3} \neq 0$, this series must DIVERGE.

(c) $\sum_{n=1}^{\infty} \frac{1}{100n}$

$\boxed{2}$ If this were to converge, then it must equal $\frac{1}{100}\sum_{n=1}^{\infty}\frac{1}{n}$. But we know $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges (harmonic series), so $\sum_{n=1}^{\infty}\frac{1}{100n}$ must also diverge.