MAT1211

Name SOLUTIONS

Quiz # 11B

ID

1. Find the area bounded by the curve $r = 2 + \sin\theta + \cos\theta$.

Since $2+\sin\theta+\cos\theta > 0$ for all θ (why?) we need not worry about self-intersections.

$\boxed{5}$

So $\text{Area} = \frac{1}{2}\int_0^{2\pi} (2+\sin\theta+\cos\theta)^2 d\theta = \frac{1}{2}\int_0^{2\pi} (4+\cos^2\theta+\sin^2\theta+4\cos\theta+4\sin\theta+2\sin\theta\cos\theta) d\theta$

$= \frac{1}{2}\int_0^{2\pi} (5+4\cos\theta+4\sin\theta+\sin 2\theta) d\theta$

$= \frac{1}{2}\left[5\theta+4\sin\theta-4\cos\theta-\frac{1}{2}\cos 2\theta\right]_{\theta=0}^{\theta=2\pi} = 5\pi$

2. Find the length of the spiral $r = \theta$ for $0 \le \theta \le 2\pi$.

$\text{Length} = \int_0^{2\pi} \sqrt{\left(\frac{dr}{d\theta}\right)^2 + r^2}\, d\theta = \int_0^{2\pi} \sqrt{1+\theta^2}\, d\theta$

$\boxed{5}$

$= \left(\frac{\theta}{2}\sqrt{1+\theta^2} + \frac{1}{2}\ln\left(\theta+\sqrt{1+\theta^2}\right)\right)\Big|_{\theta=0}^{\theta=2\pi}$

$= \pi\sqrt{1+4\pi^2} + \frac{1}{2}\ln\left(2\pi+\sqrt{1+4\pi^2}\right)$

3. Determine whether the sequence $\left\{\dfrac{\ln(1+e^{2n})}{5n}\right\}$ converges or diverges and if it converges find its limit.

$\boxed{2}$

Get $\lim_{n\to\infty} \frac{\ln(1+e^{2n})}{5n} = \lim_{n\to\infty} \frac{2e^{2n}}{(1+e^{2n})\cdot 5} = \lim_{n\to\infty} \frac{2}{5(e^{-2n}+1)} = \frac{2}{5}$ by L'Hopital's rule.

So the sequence converges to $\frac{2}{5}$.

4. Determine whether the sequence $\{2+(-1)^n/n\}$ is increasing or decreasing or not monotonic.

$\boxed{2}$ The first few terms are $1, \frac{5}{2}, \frac{7}{4} = 1, 2.5, 1.75$, so the sequence cannot be monotone.

5. Determine with reasons whether the following series are convergent or divergent:

(a) $\sum_{n=1}^{\infty} \frac{1}{e^n}$ (if convergent, find its value)

$\boxed{2}$ This is geometric with initial term $\frac{1}{e}$, common ratio $\frac{1}{e}$.
Since $0 < \frac{1}{e} < 1$ it converges. In fact, we have $\sum_{n=1}^{\infty} \frac{1}{e^n} = \frac{1/e}{1-1/e} = \frac{1}{e-1}$.

(b) $\sum_{n=1}^{\infty} \ln\left(\frac{3n+1}{2n+5}\right)$

$\boxed{2}$ Since $\lim_{n\to\infty} \ln\left(\frac{3n+1}{2n+5}\right) = \lim_{n\to\infty} \ln\left(\frac{3+1/n}{2+5/n}\right) = \ln\left(\frac{3}{2}\right) \neq 0$, this series must diverge.

(c) $\sum_{n=1}^{\infty} \frac{3}{2n}$

$\boxed{2}$ If this were to converge, then it must equal $\frac{3}{2}\sum_{n=1}^{\infty} \frac{1}{n}$. But we know $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (harmonic series), so $\sum_{n=1}^{\infty} \frac{3}{2n}$ must also diverge.