# MAT1211

Name SOLUTIONS

Quiz # 12B

ID

1. Determine with reasons whether the following series are convergent or divergent:

(a) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$ Apply the integral test.

$\boxed{4}$ Get $\int_2^{\infty} \frac{dx}{x(\ln x)^2} = \int_{\ln 2}^{\infty} \frac{du}{u^2}$ by subbing $u = \ln x$.

But $\int_{\ln 2}^{\infty} \frac{du}{u^2}$ converges, as it is of form $\int_a^{\infty} \frac{du}{u^p}$ with $p>1$.

Thus the given series also converges.

(b) $\sum_{n=1}^{\infty} \frac{1}{n+n^{3/2}}$

$\boxed{3}$ Since $n+n^{3/2} > n^{3/2}$, we have $\frac{1}{n+n^{3/2}} < \frac{1}{n^{3/2}}$. But $\sum \frac{1}{n^{3/2}}$ converges (p-series with $p=\frac{3}{2}>1$), so $\sum \frac{1}{n+n^{3/2}}$ also converges by comparison.

(c) $\sum_{n=1}^{\infty} \frac{n^3+6n^2+3n+1}{7n^3+5n^2+2n+5}$

$\boxed{3}$ We have $\lim_{n\to\infty} \frac{n^3+6n^2+3n+1}{7n^3+5n^2+2n+5} = \lim_{n\to\infty} \frac{1+\frac{6}{n}+\frac{3}{n^2}+\frac{1}{n^3}}{7+\frac{5}{n}+\frac{2}{n^2}+\frac{5}{n^3}} = \frac{1}{7} \neq 0$

Since the terms of the series don't limit to 0, the series must diverge. [This is the "test for divergence".]

(d) $\sum_{n=1}^{\infty} \frac{3n^3+5n^2-1}{2n^5+7n^3+3}$ Use limit comparison to compare with $\sum \frac{1}{n^2}$.

$\boxed{4}$ Get $\lim_{n\to\infty} \left(\frac{3n^3+5n^2-1}{2n^5+7n^3+3}\right) / \left(\frac{1}{n^2}\right) = \lim_{n\to\infty} \frac{3n^5+5n^4-1}{2n^5+7n^3+3} = \frac{3}{2}$

Since $\sum \frac{1}{n^2}$ converges, the given series also converges.

(e) $\sum_{n=1}^{\infty} \frac{1}{3^n}$ (if convergent, find its value)

$\boxed{3}$ This is geometric with initial term $\frac{1}{3}$, common ratio $\frac{1}{3}$.
Since $\frac{1}{3} < 1$, it converges, and its sum is $\frac{1/3}{1-1/3} = \frac{3}{2}$.

(f) $\sum_{n=1}^{\infty} \frac{(-1)^n}{n+\sqrt{n}}$

This is an alternating series. Clearly the sequence $\left\{ \frac{1}{n+\sqrt{n}} \right\}$
$\boxed{3}$ is decreasing and limits to 0 as $n \to \infty$, so by the
alternating series test the given series converges.