MAT1211

Name SOLUTIONS

Quiz # 13

ID

1. Determine with reasons whether the following series are convergent or divergent:

(a) $\sum_{n=1}^{\infty} \left(\frac{57n+1}{56n+2}\right)^n$

$\boxed{0}$

ROOT TEST: $\lim_{n\to\infty} \sqrt[n]{\left(\frac{57n+1}{56n+2}\right)^n} = \lim_{n\to\infty} \frac{57n+1}{56n+2} = \frac{57}{56} > 1$, so DIVERGES

(b) $\sum_{n=1}^{\infty} \frac{(2n)!}{(3n)!}$

$\boxed{0}$

RATIO TEST: $\lim_{n\to\infty} \frac{\frac{(2(n+1))!}{(3(n+1))!}}{\frac{(2n)!}{(3n)!}} = \lim_{n\to\infty} \frac{(2n+2)!\,(3n)!}{(2n)!\,(3n+3)!} = \lim_{n\to\infty} \frac{(2n+2)(2n+1)}{(3n+3)(3n+2)(3n+1)} = 0$

So the series CONVERGES

2. Determine with reasons whether the following series are absolutely convergent, conditionally convergent or divergent:

(a) $\sum_{n=1}^{\infty} \frac{(-1)^n}{3^n}$ Since $\sum_n \left|\frac{(-1)^n}{3^n}\right| = \sum_n \frac{1}{3^n}$ is geometric with common ratio $\frac{1}{3}$,

$\boxed{0}$ the given series is ABSOLUTELY CONVERGENT.

(b) $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{\sqrt{n}}$ Here $\sum_n \left|\frac{(-1)^n \ln n}{\sqrt{n}}\right| = \sum_n \frac{\ln n}{\sqrt{n}}$ diverges in comparison with $\sum \frac{1}{\sqrt{n}}$, since $\frac{\ln n}{\sqrt{n}} > \frac{1}{\sqrt{n}}$ and $\sum \frac{1}{\sqrt{n}}$ diverges. So the given series is NOT absolutely convergent.

$\boxed{0}$

But $\left\{\frac{\ln n}{\sqrt{n}}\right\}$ is eventually decreasing [CHECK!] and $\lim_{n\to\infty} \frac{\ln n}{\sqrt{n}} = 0$ [CHECK with l'Hopital], so by the A.S.T. $\sum (-1)^n \frac{\ln n}{\sqrt{n}}$ converges. This is therefore CONDITIONAL CONVERGENCE.

(c) $\sum_{n=1}^{\infty} (-1)^n \frac{n^2+3n+1}{\underbrace{n^3+2n^3}_{=3n^3}+5n+3}$

NOTE!

$\boxed{0}$ Here $\sum_n \left|(-1)^n \frac{n^2+3n+1}{3n^3+5n+3}\right| = \sum_n \frac{n^2+3n+1}{3n^3+5n+3}$. Use limit comparison to compare with $\sum_n \frac{1}{n}$. Get $\lim_{n\to\infty} \left(\frac{n^2+3n+1}{3n^3+5n+3}\right) / \left(\frac{1}{n}\right) = \lim_{n\to\infty} \frac{n^3+3n^2+n}{3n^3+5n+3} = \frac{1}{3}$.

Since $\sum \frac{1}{n}$ diverges, it follows that the given series is NOT absolutely convergent. However, $\left\{\frac{n^2+3n+1}{3n^3+5n+3}\right\}$ is eventually decreasing (why?) and limits to 0 (why?) so by the A.S.T. we conclude that the given series is CONDITIONALLY CONVERGENT

(d) $\sum_{n=1}^{\infty}(-1)^n\frac{(n!)^2 2^n}{(2n)!}$

$\boxed{0}$ RATIO TEST: $\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}(n+1)!^2\,2^{n+1}}{(2(n+1))!}}{\frac{(-1)^n (n!)^2 2^n}{(2n)!}}\right| = \lim_{n\to\infty}\frac{2(n+1)!^2(2n)!}{(2n+2)!(n!)^2} = \lim_{n\to\infty}\frac{2(n+1)^2}{(2n+2)(2n+1)} = \frac{1}{2}$

So the series CONVERGES ABSOLUTELY

(e) $\sum_{n=1}^{\infty}(-1)^n\frac{1.\,3.\,5\ldots(2n-1)}{2.\,5.\,\ldots(3n-1)}$

CHECK THIS CAREFULLY!

$\boxed{0}$ RATIO TEST: $\lim_{n\to\infty}\left|\frac{\frac{(-1)^{n+1}\,1\cdot3\cdot5\cdots(2(n+1)-1)}{2\cdot5\cdots(3(n+1)-1)}}{\frac{(-1)^n\,1\cdot3\cdot5\cdots(2n-1)}{2\cdot5\cdot8\cdots(3n-1)}}\right| = \lim_{n\to\infty}\frac{2n+2}{3n+2} = \frac{2}{3}<1$, so the series

CONVERGES ABSOLUTELY

(f) $\sum_{n=1}^{\infty}(-1)^n\left(\frac{\ln n}{n}\right)^n$

ROOT TEST: $\lim_{n\to\infty}\sqrt[n]{\left|(-1)^n\left(\frac{\ln n}{n}\right)^n\right|} = \lim_{n\to\infty}\frac{\ln n}{n} \overset{\text{L'Hopital}}{=} \lim_{n\to\infty}\frac{1/n}{1} = 0$

So the series CONVERGES ABSOLUTELY.