

MAT 1211.2 QUIZ # 1A

20SOLUTIONS

#1

$$(A) \int \cos x \sqrt{1 + \sin x} \, dx, \quad \text{let } u = 1 + \sin x$$

$$\Rightarrow du = \cos x \, dx$$

$$= \int \sqrt{u} \, du$$

$$= \frac{2}{3} u^{3/2} + C$$

$$= \frac{2}{3} (1 + \sin x)^{3/2} + C$$

/4

$$(B) \int \frac{e^x}{1 + e^x} \, dx, \quad \text{let } u = 1 + e^x$$

$$\Rightarrow du = e^x \, dx$$

$$= \int \frac{du}{u}$$

$$= \ln |u| + C$$

/4

$$= \ln |1 + e^x| + C \quad \left\{ \text{or } \ln(1 + e^x) + C, \text{ since } 1 + e^x > 0 \right\}$$

$$(C) \int \frac{1}{x \ln x} \, dx, \quad \text{let } u = \ln x$$

$$\Rightarrow du = \frac{1}{x} \, dx$$

$$= \int \frac{du}{u}$$

$$= \ln |u| + C$$

/4

$$= \ln |\ln x| + C$$

$$\textcircled{1} \int x^9 \sqrt[3]{1+x^5} dx \quad \text{let } u = 1+x^5$$

$$\Rightarrow du = 5x^4 dx$$

$$= \int \frac{1}{5} (u-1) u^{1/3} du$$

$$= \frac{1}{5} \int (u^{4/3} - u^{1/3}) du$$

$$= \frac{1}{5} \left(\frac{3}{7} u^{7/3} - \frac{3}{4} u^{4/3} \right) + C \quad /5$$

$$= \frac{3}{35} (1+x^5)^{7/3} - \frac{3}{20} (1+x^5)^{4/3} + C$$

#2 Evaluate $\int_{-9}^9 \frac{x^5 - x^3}{x^6 + x^4 + 9x^2 + 8} dx$ using symmetry.

Sol'n: Let $f(x) = \frac{x^5 - x^3}{x^6 + x^4 + 9x^2 + 8}$.

$$\text{Then } f(-x) = \frac{(-x)^5 - (-x)^3}{(-x)^6 + (-x)^4 + 9(-x)^2 + 8}$$

$$= - \left(\frac{x^5 - x^3}{x^6 + x^4 + 9x^2 + 8} \right)$$

$$= -f(x). \quad /3$$

So $f(-x) = -f(x)$, meaning f is an odd function.

$$\text{Thus } \int_{-9}^9 \frac{x^5 - x^3}{x^6 + x^4 + 9x^2 + 8} dx = 0.$$