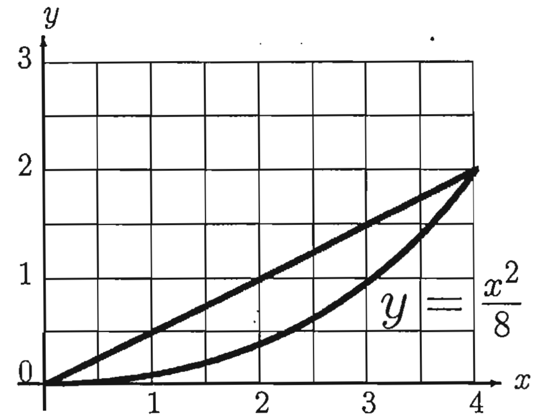
MAT1211

Name SOLUTIONS

Quiz # 3A

ID ____________________

1. Give the definite integral that results when finding the volume generated by revolving the region shown in the diagram about the line $x = -1$ when using the cylindrical shell method (2nd method). Do NOT evaluate the integral.



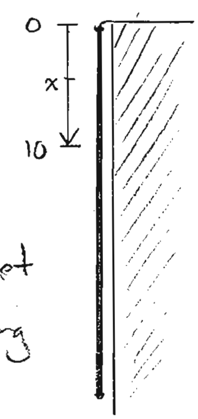
The upper line is clearly $y = \frac{x}{2}$.

Thus the desired volume is

$$2\pi \int_0^4 (x+1)\left(\frac{x}{2} - \frac{x^2}{8}\right) dx$$

$\boxed{5}$

2. A uniform cable hanging over the edge of a tall building is 80 ft long and weighs 160 lbs. How much work is required to pull 10 ft of the cable to the top?

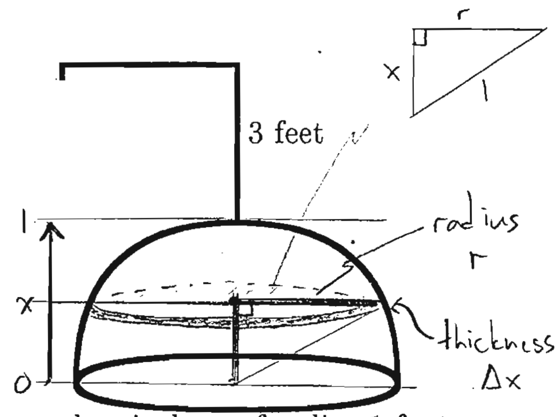


We use the coordinate system shown in the diagram. The small piece of cable between x and $x + \Delta x$ has length Δx and thus weight $2\Delta x$ lbs., since the density of the cable is $160/80 = 2$ lbs/ft. This bit of cable will rise x feet for $0 \le x \le 10$ (requiring work $2x\Delta x$), and 10 feet for $10 \le x \le 80$ (requiring work $10 \cdot 2\Delta x$).

$\boxed{5}$

Thus the total work is $\int_0^{10} 2x\,dx + \int_{10}^{80} 20\,dx = x^2\Big|_0^{10} + 20x\Big|_{10}^{80} = 100 + 1400 = 1500$ ft-lbs.

3. Find the work required to pump all the liquid out of the outlet as shown in the diagram. Assume the tank is full and the liquid weighs ω pounds per cubic foot.



hemisphere of radius 1 foot

Consider the coordinate system shown in the diagram. The slab of water at height x and of thickness Δx has radius r, where $x^2+r^2=1^2$. Its weight is approximately (volume)·(density) $\approx (\pi r^2 \Delta x)(w) = \pi w(1-x^2)\Delta x$. This water must rise $(1-x)+3 = 4-x$ feet.

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The total work done is then:

$$\int_0^1 \pi w(1-x^2)(4-x)\,dx = \pi w\int_0^1 (4-x-4x^2+x^3)\,dx = \frac{29}{12}\pi w \text{ ft-lbs.}$$

4. Find the average value of the function $f(x) = e^{2x}$ on the interval $[0, \frac{1}{2}]$.

By definition, the average value is $\frac{1}{1/2 - 0}\int_0^{1/2} e^{2x}\,dx = 2\cdot\left[\frac{1}{2}e^{2x}\right]_{x=0}^{x=1/2}$

$= 2\left(\frac{1}{2}e^1 - \frac{1}{2}e^0\right)$

$= e-1.$

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