MAT1211

Name SOLUTIONS

Quiz # 5A

ID

1. Find the following antiderivatives:

(a) $\int \frac{x^3}{\sqrt{4-x^2}}dx$ SET $x = 2\sin\theta$, so $dx = 2\cos\theta\, d\theta$; $\sqrt{4-x^2} = \sqrt{4\cos^2\theta} = 2\cos\theta$

$= \int \frac{2^3 \sin^3\theta}{2\cos\theta} \cdot 2\cos\theta\, d\theta$

$\boxed{5}$ $= 8\int \sin^3\theta\, d\theta$

$= 8\int \sin^2\theta \cdot \sin\theta\, d\theta$

$= 8\int (1-\cos^2\theta)\sin\theta\, d\theta$

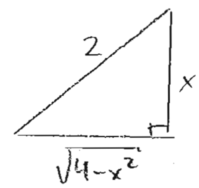
$= -8\int (1-u^2)\,du$, where $u = \cos\theta$

$= -8u + \frac{8}{3}u^3 + C$

$= -8\cos\theta + \frac{8}{3}\cos^3\theta + C$

But $x = 2\sin\theta \Rightarrow \sin\theta = \frac{x}{2}$

Hence $\cos\theta = \frac{\sqrt{4-x^2}}{2}$



So $\int \frac{x^3}{\sqrt{4-x^2}}dx = -\sqrt{4-x^2} + \frac{1}{3}(4-x^2)^{3/2} + C$

$\left[= -\frac{1}{3}\sqrt{4-x^2}\,(x^2+8) + C \right]$

(b) $\int \frac{x^3}{\sqrt{x^2+4}}dx$ SET $x = 2\tan\theta \Rightarrow \begin{cases} dx = 2\sec^2\theta\, d\theta \\ \sqrt{x^2+4} = 2\sec\theta \end{cases}$

$= \int \frac{2^3\tan^3\theta}{2\sec\theta} \cdot 2\sec^2\theta\, d\theta$

$\boxed{5}$ $= 8\int \tan^3\theta \sec\theta\, d\theta$

$= 8\int \tan^2\theta \cdot \sec\theta\tan\theta\, d\theta$

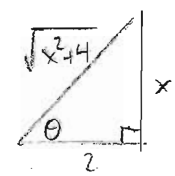
$= 8\int (\sec^2\theta - 1)\cdot \sec\theta\tan\theta\, d\theta$

$= 8\int (u^2-1)\,du$, where $u = \sec\theta$

$= \frac{8}{3}u^3 - 8u + C = \frac{8}{3}\sec^3 u - 8\sec u + C$

But $x = 2\tan\theta \Rightarrow \tan\theta = \frac{x}{2}$

$\Rightarrow \sec\theta = \frac{\sqrt{x^2+4}}{2}$



So $\int \frac{x^3}{\sqrt{x^2+4}}dx = \frac{1}{3}(x^2+4)^{3/2} - \sqrt{x^2+4} + C$

$\left[= \frac{1}{3}\sqrt{x^2+4}\,(x^2-8) + C \right]$

(c) $\int \frac{1}{x^2\sqrt{x^2-25}}dx$ SET $x = 5\sec\theta$, so $dx = 5\sec\theta\tan\theta$ and $\sqrt{x^2-25} = 5\tan\theta$.

$= \int \frac{5\sec\theta\tan\theta}{5^2\sec^2\theta \cdot 5\tan\theta}\, d\theta$

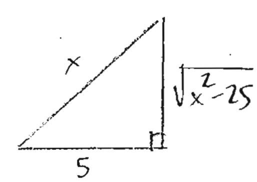
$\boxed{5}$ $= \frac{1}{25}\int \frac{d\theta}{\sec\theta}$

$= \frac{1}{25}\int \cos\theta\, d\theta$

$= \frac{1}{25}\sin\theta + C$

But $x = 5\sec\theta$, so $\sec\theta = \frac{x}{5}$

and hence $\sin\theta = \frac{\sqrt{x^2-25}}{x}$



Thus $\int \frac{dx}{x^2\sqrt{x^2-25}} = \frac{\sqrt{x^2-25}}{25x} + C$

(d) $\int xe^{-2x}\,dx$ (just for practice)

Use integration by parts with $u = x$, $du = dx$
$dv = e^{-2x}dx$, $v = -\frac{1}{2}e^{-2x}$

$\boxed{5}$

Get $\int xe^{-2x}\,dx = -\frac{1}{2}xe^{-2x} - \int\left(-\frac{1}{2}e^{-2x}\right)dx$

$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$

$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C \qquad \left[= -\frac{1}{4}e^{-2x}(2x+1) + C\right].$$