MAT1211

Quiz # 5B

Name SOLUTIONS

ID ____________

1. Find the following antiderivatives:

(a) $\displaystyle\int \frac{1}{x^2\sqrt{9-x^2}}dx$ SET $x = 3\sin\theta \Rightarrow dx = 3\cos\theta\, d\theta$, $\sqrt{9-x^2} = \sqrt{9\cos^2\theta} = 3\cos\theta$.

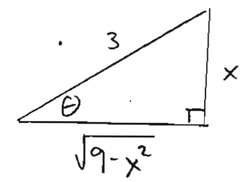
$$= \int \frac{3\cos\theta}{9\sin^2\theta \cdot 3\cos\theta}d\theta$$

$$\boxed{5} = \frac{1}{9}\int \frac{d\theta}{\sin^2\theta}$$

$$= \frac{1}{9}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{9}\cot\theta + C$$

But $x = 3\sin\theta \Rightarrow \sin\theta = \frac{x}{3}$.



Thus $\cot\theta = \dfrac{\sqrt{9-x^2}}{x}$

So $\displaystyle\int \frac{1}{x^2\sqrt{9-x^2}}dx = -\frac{\sqrt{9-x^2}}{9x} + C$.

(b) $\displaystyle\int \frac{dx}{(x^2+4)^2}$ SET $x = 2\tan\theta \Rightarrow dx = 2\sec^2\theta\, d\theta$, $x^2+4 = 4\tan^2\theta + 4 = 4\sec^2\theta$

$$= \int \frac{2\sec^2\theta}{(4\sec^2\theta)^2}d\theta$$

$$\boxed{5} = \frac{1}{8}\int \frac{d\theta}{\sec^2\theta}$$

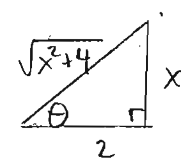
$$= \frac{1}{8}\int \cos^2\theta\, d\theta$$

$$= \frac{1}{8}\int \frac{1}{2}(1+\cos 2\theta)d\theta$$

$$= \frac{\theta}{16} + \frac{1}{32}\sin 2\theta + C = \frac{\theta}{16} + \frac{1}{16}\sin\theta\cos\theta + C$$

But $x = 2\tan\theta \Rightarrow \tan\theta = \frac{x}{2} \Rightarrow \theta = \tan^{-1}\left(\frac{x}{2}\right)$

Thus $\sin\theta = \dfrac{x}{\sqrt{4+x^2}}$



and $\cos\theta = \dfrac{2}{\sqrt{4+x^2}}$,

So $\cos\theta\sin\theta = \dfrac{2x}{4+x^2}$

Hence $\displaystyle\int \frac{dx}{(x^2+4)^2} = \frac{1}{16}\tan^{-1}\left(\frac{x}{2}\right) + \frac{x}{8(4+x^2)} + C$.

(c) $\displaystyle\int \frac{x^3}{\sqrt{x^2-9}}dx$ SET $x = 3\sec\theta$, so $dx = 3\sec\theta\tan\theta\, d\theta$, $\sqrt{x^2-9} = \sqrt{9\tan^2\theta} = 3\tan\theta$.

$$= \int \frac{3^3\sec^3\theta}{3\tan\theta}\cdot 3\sec\theta\tan\theta\, d\theta$$

$$\boxed{5} = 27\int \sec^4\theta\, d\theta$$

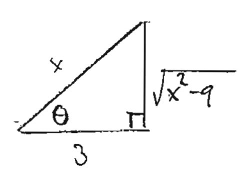
$$= 27\int \sec^2\theta\cdot\sec^2\theta\, d\theta$$

$$= 27\int (1+\tan^2\theta)\sec^2\theta\, d\theta$$

$$= 27\int (1+u^2)du,\quad u = \tan\theta$$

$$= 27u + 9u^3 + C = 27\tan\theta + 9\tan^3\theta + C$$

But $x = 3\sec\theta \Rightarrow \sec\theta = \frac{x}{3}$



Thus $\tan\theta = \dfrac{\sqrt{x^2-9}}{3}$.

So $\displaystyle\int \frac{x^3}{\sqrt{x^2-9}}dx = 9\sqrt{x^2-9} + \frac{1}{3}(x^2-9)^{3/2} + C$

$$\left[= \sqrt{x^2-9}\left(\frac{x^2}{3}+6\right) + C \right]$$

(d) $\int xe^{-2x}\,dx$ (just for practice)

Use integration by parts with $u = x$, $du = dx$; $dv = e^{-2x}\,dx$, $v = -\frac{1}{2}e^{-2x}$

$\boxed{5}$

Get
$$\int xe^{-2x}\,dx = -\frac{1}{2}xe^{-2x} - \int\left(-\frac{1}{2}e^{-2x}\right)dx$$
$$= -\frac{1}{2}xe^{-2x} + \frac{1}{2}\int e^{-2x}\,dx$$
$$= -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$$
$$\left[= -\frac{1}{4}e^{-2x}(2x+1) + C\right].$$