MAT 1211: Quiz #6B

SOLUTIONS

PROBLEM 1: $\int \frac{x^3}{x^2+x-6}\,dx$

SOLUTION: First divide to get $\frac{x^3}{x^2+x-6} = x-1 + \frac{7x-6}{x^2+x-6}$.

Then let $\frac{7x-6}{x^2+x-6} = \frac{7x-6}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$.

This gives $7x-6 = A(x-2) + B(x+3)$, so $x=2 \Rightarrow 8=5B \Rightarrow B=\frac{8}{5}$,
and $x=-3 \Rightarrow -27=-5A \Rightarrow A = 27/5$.

Hence $\int \frac{x^3}{x^2+x-6}\,dx = \int \left(x-1 + \frac{27}{5}\cdot\frac{1}{x+3} + \frac{8}{5}\cdot\frac{1}{x-2}\right)dx$

$$= \frac{1}{2}x^2 - x + \frac{27}{5}\ln|x+3| + \frac{8}{5}\ln|x-2| + C.$$

PROBLEM 2. $\int \frac{x-2}{x^3-x^2}\,dx$

SOLUTION: Let $\frac{x-2}{x^3-x^2} = \frac{x-2}{x^2(x-1)} = \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-1}$.

This gives $x-2 = Ax(x-1) + B(x-1) + Cx^2$.

From here, $x=0 \Rightarrow -2=-B \Rightarrow B=2$, $x=1 \Rightarrow -1=C$.
Compare coefficients of x^2 to get $0 = A+C \Rightarrow A=1$.

Thus $\int \frac{x-2}{x^3-x^2}\,dx = \int \left(\frac{1}{x} + \frac{2}{x^2} - \frac{1}{x-1}\right)dx$

$$= \ln|x| - \frac{2}{x} - \ln|x-1| + C$$

PROBLEM 3: $\int \frac{4x^2-2x}{(x^2+1)(x-1)}\,dx$

SOLUTION: Let $\frac{4x^2-2x}{(x^2+1)(x-1)} = \frac{Ax+B}{x^2+1} + \frac{C}{x-1}$, so $4x^2-2x = (Ax+B)(x-1) + C(x^2+1)$

Then $x=1 \Rightarrow 2 = 2C \Rightarrow C = 1$, $x=0 \Rightarrow 0 = -B+C \Rightarrow B = 1$.
Compare coefficients of x^2 to get $4 = A + C \Rightarrow A = 3$.

Thus $\int \frac{4x^2-2x}{(x^2+1)(x-1)}\,dx = \int \left(\frac{3x+1}{x^2+1} + \frac{1}{x-1} \right) dx$

$$= 3\int \frac{x}{x^2+1}\,dx + \int \frac{dx}{x^2+1} + \int \frac{dx}{x-1}$$

$$= \frac{3}{2}\ln(x^2+1) + \tan^{-1}x + \ln|x-1| + C.$$

PROBLEM 4: $\int \frac{e^{2x}}{e^{2x}+e^x-2}\,dx$

SOLUTION: Substitute $u = e^x$, so $du = e^x\,dx$.

This gives $\int \frac{e^{2x}}{e^{2x}+e^x-2}\,dx = \int \frac{e^x \cdot e^x\,dx}{e^{2x}+e^x-2} = \int \frac{u}{u^2+u-2}\,du$.

Now let $\frac{u}{u^2+u-2} = \frac{u}{(u+2)(u-1)} = \frac{A}{u+2} + \frac{B}{u-1}$

This gives $u = A(u-1) + B(u+2)$, so $u=1 \Rightarrow 1 = 3B \Rightarrow B = 1/3$,
and $u=-2 \Rightarrow -2 = -3A \Rightarrow A = 2/3$

Hence $\int \frac{u}{u^2+u-2}\,du = \int \left(\frac{2}{3} \cdot \frac{1}{u+2} + \frac{1}{3} \cdot \frac{1}{u-1} \right) du = \frac{2}{3}\ln|u+2| + \frac{1}{3}\ln|u-1| + C.$

Therefore $\int \frac{e^{2x}}{e^{2x}+e^x-2}\,dx = \frac{2}{3}\ln(e^x+2) + \frac{1}{3}\ln(e^x-1) + C$