MAT1211

Name SOLUTIONS

Quiz # 7B

ID

1. Approximate $\int_0^2 \frac{dx}{x+1}$ by:

(a) the trapezoidal rule with $n = 4$;

(b) Simpson's rule with $n = 4$.

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a) Let $f(x) = \frac{1}{x+1}$. Here $\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$\boxed{5}$ Thus $\int_0^2 \frac{dx}{x+1} \approx \frac{1}{4}\left(f(0) + 2f(0.5) + 2f(1.0) + 2f(1.5) + f(2)\right) = \frac{67}{60}$

b) Again $\Delta x = \frac{1}{2}$.

(b)

Thus $\int_0^2 \frac{dx}{x+1} \approx \frac{1}{6}\left(f(0) + 4f(0.5) + 2f(1) + 4f(1.5) + f(2)\right) = \frac{11}{10}$

2. Find the value of the following improper integrals or show they do not converge:

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(a) $\int_0^\infty \frac{dx}{x^2+1}$

$\boxed{5}$ We have $\int_0^\infty \frac{dx}{x^2+1} = \lim_{t\to\infty} \int_0^t \frac{dx}{x^2+1} = \lim_{t\to\infty} \tan^{-1}(x)\Big|_{x=0}^{x=t}$

$= \lim_{t\to\infty}\left(\tan^{-1}(t) - 0\right) = \frac{\pi}{2}$

3.

So the integral converges to $\frac{\pi}{2}$.

(b) $\int_1^2 \frac{dx}{\sqrt[3]{x-1}}$

$\boxed{5}$ Get $\int_1^2 \frac{dx}{\sqrt[3]{x-1}} = \lim_{t\to 1^+} \int_t^2 (x-1)^{-1/3}\,dx = \lim_{t\to 1^+} \frac{3}{2}(x-1)^{2/3}\Big|_{x=t}^{x=2}$

$= \lim_{t\to 1^+}\left(\frac{3}{2} - \frac{3}{2}(t-1)^{2/3}\right) = \frac{3}{2}$

So the integral converges to $\frac{3}{2}$.

3. Find the length of the curve $y = \dfrac{x^5}{10} + \dfrac{1}{6x^3}$ from $x = 1$ to $x = 2$.

$\boxed{5}$

The desired length is $\displaystyle\int_1^2 \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx = \int_1^2 \sqrt{1+\left(\frac{x^4}{2}-\frac{1}{2x^4}\right)^2}\,dx$

$$= \int_1^2 \sqrt{\frac{x^8}{4}+\frac{1}{2}+\frac{1}{4x^8}}\,dx$$

$$= \int_1^2 \sqrt{\left(\frac{x^4}{2}+\frac{1}{2x^4}\right)^2}\,dx$$

$$= \int_1^2 \left(\frac{x^4}{2}+\frac{1}{2x^4}\right)dx$$

$$= \left(\frac{x^5}{10}-\frac{1}{6x^3}\right)\Big|_1^2 = \frac{779}{240}$$