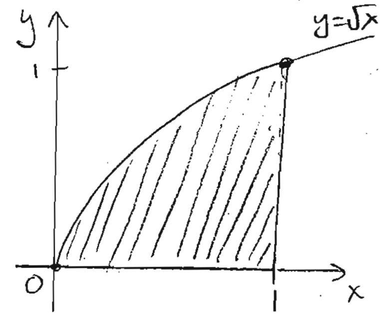
MAT1211

Name SOLUTIONS

Quiz # 8A

ID ______

1. Find the area of the surface of revolution generated by revolving the area bounded by $y = \sqrt{x}$ between $x = 0$ and $x = 1$ about the x-axis.



We use $\text{Area} = 2\pi \int y\, ds$, with $ds = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$\boxed{7}$

Here $x = y^2$, so $1 + \left(\frac{dx}{dy}\right)^2 = 1 + (2y)^2 = 1 + 4y^2$

Thus $\text{Area} = 2\pi \int_0^1 y\sqrt{1+4y^2}\, dy$

$$= 2\pi \cdot \tfrac{1}{12}(1+4y^2)^{3/2}\Big]_0^1$$

$$= 2\pi\left[\frac{5\sqrt{5}}{12} - \frac{1}{12}\right]$$

$$= \frac{\pi}{6}\left(5\sqrt{5} - 1\right)$$

[OR $\text{Area} = 2\pi \int_0^1 \underbrace{\sqrt{x}}_{y} \sqrt{1 + \underbrace{\left(\frac{1}{2\sqrt{x}}\right)^2}_{\left(\frac{dy}{dx}\right)^2}}\, dx$

$\vdots$

$=$ same result]

2. Find the complete solution to the differential equation $\dfrac{dy}{dx} = \dfrac{\sqrt{1-y^2}}{x}$.

$\boxed{7}$

Separate variables to get $\dfrac{dy}{\sqrt{1-y^2}} = \dfrac{dx}{x} \Rightarrow \displaystyle\int \frac{dy}{\sqrt{1-y^2}} = \int \frac{dx}{x}$

$\Rightarrow \sin^{-1}(y) = \ln|x| + C$

$\Rightarrow y = \sin(\ln|x| + C)$

[<u>Note</u>: We should also include the constant solutions $y(x) = \pm 1$]

3. Find the solution to the differential equation $\frac{dy}{dx} = 2xy^3(2x^2+1), \quad y(1) = 1.$

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Separate variables to get $\frac{dy}{y^3} = 2x(2x^2+1)dx \Rightarrow \int \frac{dy}{y^3} = \int 2x(2x^2+1)dx$

$\Rightarrow -\frac{1}{2y^2} = x^4 + x^2 + C$

Now $y(1) = 1 \Rightarrow -\frac{1}{2} = 1^4 + 1^2 + C \Rightarrow C = -\frac{5}{2}$.

So $y^2 = -\frac{1}{2}\left(\frac{1}{x^4+x^2-\frac{5}{2}}\right) = \frac{1}{5-2x^2-2x^4} \Longrightarrow y = \frac{1}{\sqrt{5-2x^2-2x^4}}$

Here we have thrown out the negative root because of the condition $y(1) = 1$.