MAT1211

Name SOLUTION

Quiz # 8B

ID ______________________

1. Find the area of the surface of revolution generated by revolving the area bounded by $y = x^2 - \frac{1}{8}\ln x$ between $x = 0$ and $x = 2$ about the y-axis.

We apply the formula $\text{Area} = 2\pi \int x \, ds$, where $ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$

$\boxed{7}$

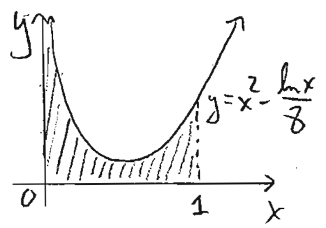
Here $1 + \left(\frac{dy}{dx}\right)^2 = 1 + \left(2x - \frac{1}{8x}\right)^2 = \frac{1}{2} + 4x^2 - \frac{1}{64x^2} = \left(2x + \frac{1}{8x}\right)^2$

Thus $\text{Area} = 2\pi \int_0^2 x\left(2x + \frac{1}{8x}\right) dx = 2\pi \int_0^2 \left(2x^2 + \frac{1}{8}\right) dx$

$= 2\pi \left(\frac{2}{3}x^3 + \frac{x}{8}\right)\Big|_0^2$

$= \frac{67\pi}{6}$

NOTE:



There is a discontinuity in this function that we really should be thinking about, at $x = 0$.

2. Find the complete solution to the differential equation $\frac{dy}{dx} = \frac{1}{(x^2+1)(y^2+1)}$. (do not simplify)

Separate variables to get: $(y^2+1)\,dy = \frac{dx}{x^2+1} \implies \int (y^2+1)\,dy = \int \frac{dx}{x^2+1}$

$\boxed{7}$

$\implies \frac{1}{3}y^3 + y = \tan^{-1} x + C$

[This is about the best we can do here – solving for $y(x)$ is hard]

3. Consider a breed of rabbits whose population $P(t)$ satisfies the initial value problem $\frac{dP}{dt} = kP^2$, $P(0) = P_0$, where k is a positive constant. Find P as a function of t.

$\boxed{7}$

Separate variables to get $\frac{dP}{P^2} = k\,dt \Rightarrow \int \frac{dP}{P^2} = \int k\,dt$

$$\Rightarrow -\frac{1}{P} = kt + C$$

$$\Rightarrow P = \frac{-1}{kt+C}$$

Now $P(0) = P_0 \Rightarrow P_0 = -1/C \Rightarrow C = -1/P_0$. [Note: This forces $P_0 \neq 0$]

So $P(t) = \frac{-1}{kt - 1/P_0} = \frac{P_0}{1-kP_0 t}$

[Note that $P(t)=0$ does solve the DE, yet we said $P_0 \neq 0$. What's going on here?]