MAT1211

Name SOLUTIONS

Quiz # 9A

ID ______________________

1. Assume that the rate of growth of a population of fruit flies is propotional to the size the population at each instant of time. If 200 fruit flies are present initially and 700 are present after 20 days, how many will be present after 30 days?

Let $P(t)$ be the population after t days. Then $P(t) = 200e^{kt}$,

$\boxed{7}$ since we have exponential growth.

We know $P(20) = 700$, so $700 = 200e^{20k} \Rightarrow e^{20k} = \frac{7}{2}$.

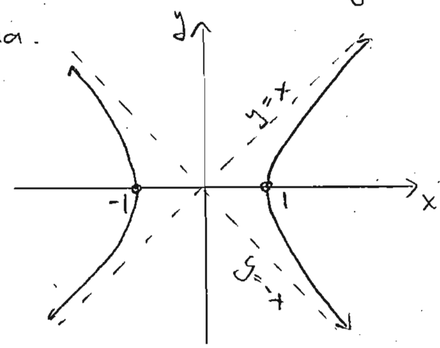
We want $P(30) = 200e^{30k} = 200\left(e^{20k}\right)^{3/2} = 200\left(\frac{7}{2}\right)^{3/2} = 700\sqrt{\frac{7}{2}}$.

2. Sketch the curve represented by each of the following sets of parametric equations by eliminating the parameter:

(a) $x = \sec t, \quad y = \tan t$

Since $\tan^2 t + 1 = \sec^2 t$, get $x^2 - y^2 = 1$. This is a rectangular hyperbola.

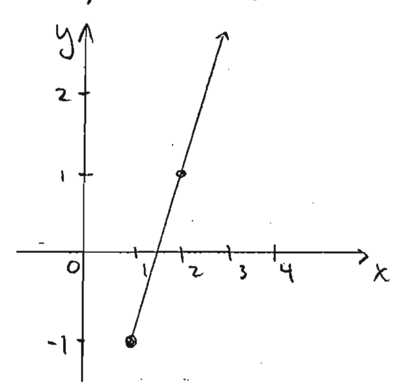
$\boxed{6}$



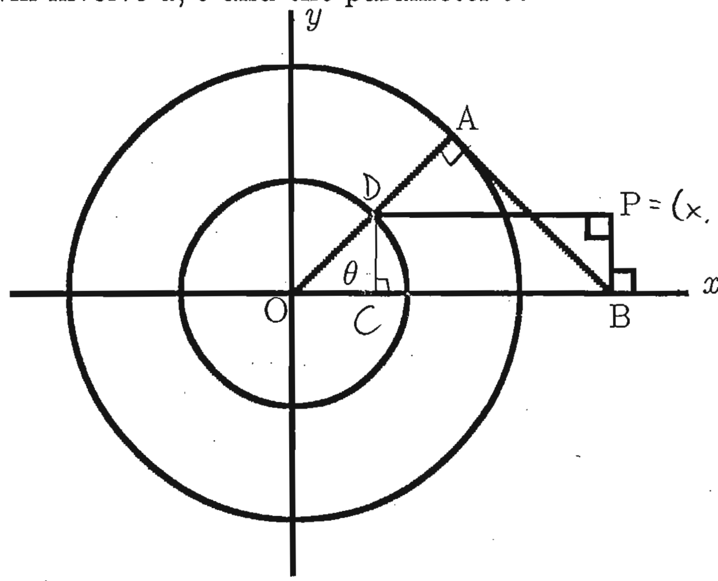
(b) $x = t^2 + 1, \quad y = 2t^2 - 1$

We have $y = 2t^2 - 1 = 2(x-1) - 1 = 2x - 3$.
This is obviously a line, but note that $x = t^2 + 1$ forces $x \geq 1$.

So the curve is actually the "half-line" pictured to the right.



3. Find the parametric equations for the set of all points P determined as shown in the diagram using θ as the parameter. The line AB is tangent to the large circle. The large circle has radius a and the small circle has radius b. Your answer will involve a, b and the parameter θ.



Let $P = (x, y)$.

Let points C, D be as in the diagram (so $\angle OCD = 90^\circ$).

Then $y = |DC| = |OD| \sin\theta$
$= b\sin\theta$.

$\boxed{7}$

Since AB is a tangent and OA a radius, we know $\angle OAB = 90^\circ$

Hence $\cos\theta = \frac{|OA|}{|OB|} = \frac{a}{x}$, and so $x = a\sec\theta$.

Altogether, $(x,y) = (a\sec\theta, b\sin\theta)$.