

## Math 1211 – Quiz #10

Fall 2011

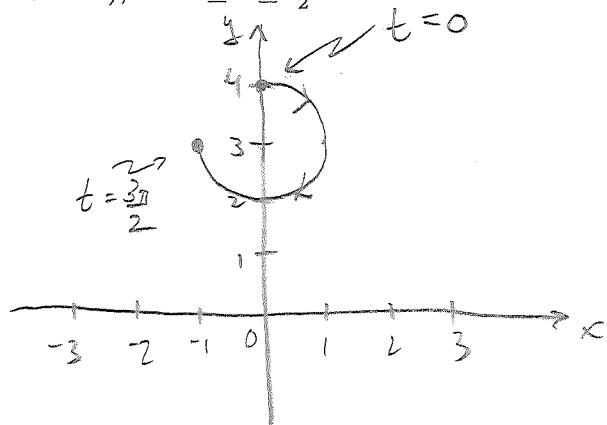
Name: SOLUTIONS

A#:

1. (a) Sketch the parametric curve  $(x, y) = (\sin t, 3 + \cos t)$ , for  $0 \leq t \leq \frac{3\pi}{2}$ .

Note that  $x^2 + (y - 3)^2 = 1$ .

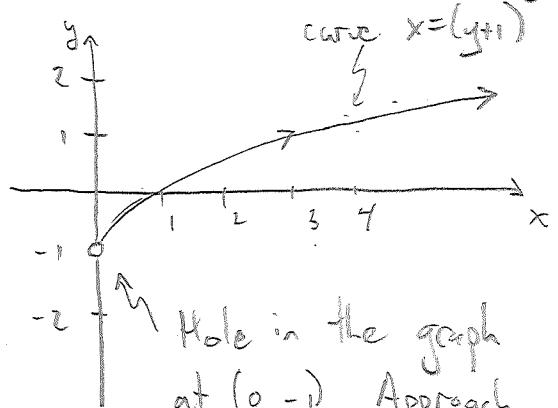
So this is part of a circle  
of radius 1 centred at  
(0, 3)



- (b) Eliminate the parameter to find the Cartesian equation of the curve  $(x, y) = (e^{2t}, e^t - 1)$ , for  $t \in \mathbb{R}$ . Use this to sketch the curve.

Here we have  $x = (y+1)^2$ . But for  $t \in \mathbb{R}$   
we have  $e^{2t} > 0$  and  $e^t - 1 > -1$ , since  $e^x$  is  
always positive

Thus  $x > 0$  and  $y > -1$ .



2. Consider the parametric curve  $(x, y) = (t^3 - 12t, t^2 - 1)$ .

- (a) Find the equation of the tangent line at the point where  $t = 1$ .

$$\frac{dx}{dt} = 3t^2 - 12, \quad \frac{dy}{dt} = 2t \quad \Rightarrow \quad \frac{dy}{dx} = \frac{2t}{3t^2 - 12}$$

$$\text{So } \left. \frac{dy}{dx} \right|_{t=1} = -\frac{2}{9}.$$

When  $t = 1$  we have  $(x, y) = (1^3 - 12 \cdot 1, 1^2 - 1) = (-11, 0)$ .

So the tangent line at this point has eqn

$$y = -\frac{2}{9}(x + 11)$$

- (b) Find all points  $(x, y)$  where the tangent is horizontal, and all points where the tangent is vertical.

From (a) we have  $\frac{dx}{dt} = 3t^2 - 12$ ,  $\frac{dy}{dt} = 2t$

$$\begin{aligned} \text{So } \frac{dx}{dt} = 0 &\Leftrightarrow 3t^2 - 12 = 0, \quad \frac{dy}{dt} = 0 \Leftrightarrow 2t = 0 \\ &\Leftrightarrow t^2 = 4 \qquad \qquad \qquad \Leftrightarrow t = 0 \\ &\Leftrightarrow t = \pm 2 \end{aligned}$$

$$\text{At } t = 2, (x, y) = (2^3 - 12 \cdot 2, 2^2 - 1) = (-16, 3)$$

$$\text{At } t = -2, (x, y) = ((-2)^3 - 12(-2), (-2)^2 - 1) = (16, 3)$$

$$\text{At } t = 0, (x, y) = (0^3 - 12 \cdot 0, 0^2 - 1) = (0, -1)$$

So we have horizontal tangents at  $(x, y) = (\pm 16, 3)$   
and a vertical tangent at  $(x, y) = (0, -1)$

- (c) Find a formula for  $\frac{d^2y}{dx^2}$  in terms of  $t$ .

$$\begin{aligned} \frac{d^2y}{dx^2} &= \frac{\frac{d}{dt}\left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{d}{dt}\left(\frac{2t}{3t^2 - 12}\right)}{3t^2 - 12} \\ &= \frac{2(3t^2 - 12) - 6t(2t)}{(3t^2 - 12)^3} \\ &= \boxed{\frac{-6(t^2 + 4)}{(3t^2 - 12)^3}} \end{aligned}$$

3. (a) Find the  $(x, y)$  coordinates of the points  $(r, \theta) = (2, -\frac{\pi}{4})$  and  $(r, \theta) = (-3, \frac{\pi}{3})$ .

$$(r, \theta) = (2, -\frac{\pi}{4}) \Rightarrow (x, y) = (2 \cos(-\frac{\pi}{4}), 2 \sin(-\frac{\pi}{4})) = \boxed{(\sqrt{2}, -\sqrt{2})}$$

$$(r, \theta) = (-3, \frac{\pi}{3}) \Rightarrow (x, y) = (-3 \cos(\frac{\pi}{3}), -3 \sin(\frac{\pi}{3})) = \boxed{(-\frac{3}{2}, -\frac{3\sqrt{3}}{2})}$$

- (b) Find three distinct (but equivalent) pairs of polar coordinates  $(r, \theta)$  of the point  $(x, y) = (2, 2\sqrt{3})$ .

$$r = \sqrt{x^2 + y^2} = \sqrt{2^2 + (2\sqrt{3})^2} = \sqrt{4 + 12} = 4$$

$$\theta = \tan^{-1}(\frac{y}{x}) = \tan^{-1}(\sqrt{3}) = \frac{\pi}{3}.$$

$$\text{So } (r, \theta) = \boxed{(4, \frac{\pi}{3})} = \boxed{(4, \frac{7\pi}{3})} = \boxed{(4, -\frac{5\pi}{3})}$$

INFINITELY  
MANY  
POSSIBLE CORRECT  
ANSWERS!