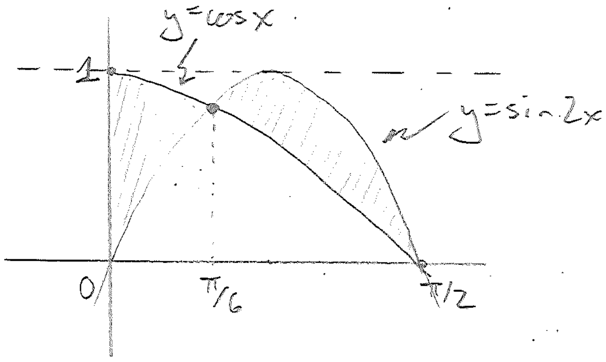
Math 1211 – Quiz #3 Fall 2011

Name: SOLUTIONS	A#:

For each of the following questions, **begin by drawing a relevant diagram** and then compute the required area or volume.

1. Find the area bounded between $y = \cos x$ and $y = \sin 2x$ on the interval $0 \le x \le \frac{\pi}{2}$. Begin by sketching this region, being sure to appropriately label your diagram.



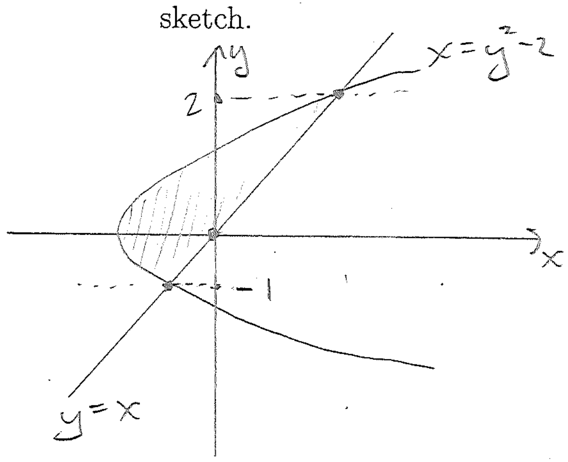
Find int. point:

$$\cos x = \sin 2x \iff \cos x = 2\sin x\cos x$$
$$\iff \cos x(1-2\sin x) = 0$$
$$\iff \cos x = 0 \text{ OR } \sin x = \tfrac{1}{2}$$

For $x \in [0, \frac{\pi}{2}]$, get only $x = \frac{\pi}{2}$ and $x = \frac{\pi}{6}$

Then
$$\text{AREA} = \int_0^{\pi/6} (\cos x - \sin 2x)\,dx + \int_{\pi/6}^{\pi/2} (\sin 2x - \cos x)\,dx$$
$$= \left(\sin x + \tfrac{1}{2}\cos 2x\right)\Big]_0^{\pi/6} + \left(-\tfrac{1}{2}\cos 2x - \sin x\right)\Big]_{\pi/6}^{\pi/2}$$
$$= \left(\tfrac{1}{2}+\tfrac{1}{4}\right) - \left(0+\tfrac{1}{2}\right) + \left(\tfrac{1}{2}-1\right) - \left(-\tfrac{1}{4}-\tfrac{1}{2}\right)$$
$$= \boxed{\tfrac{1}{2}}$$

2. Find the area bounded between the curves $y^2 = x+2$ and $y = x$. Begin with a relevant sketch.



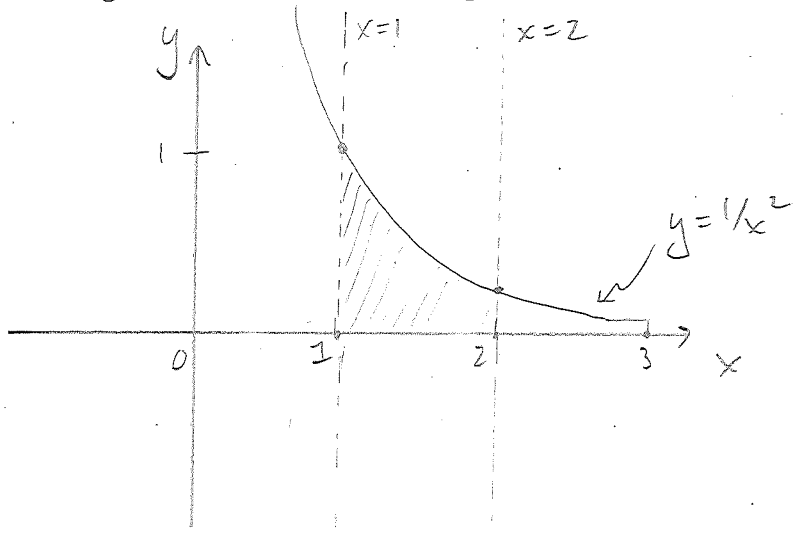
Get y-coord of int. points:

$$y^2 = y+2 \iff y^2 - y - 2 = 0$$
$$\iff (y-2)(y+1) = 0$$
$$\iff y = 2 \text{ OR } y = -1$$

So
$$\text{AREA} = \int_{-1}^{2} \left(y - (y^2-2)\right)dy$$
$$= \left(\tfrac{1}{2}y^2 - \tfrac{1}{3}y^3 + 2y\right)\Big]_{-1}^{2}$$
$$= \tfrac{10}{3} - \left(-\tfrac{7}{6}\right)$$
$$= \tfrac{27}{6}$$
$$= \boxed{\tfrac{9}{2}}$$

3. Let $\mathcal{R}$ be the region bounded between the curves $y = 1/x^2$, $y = 0$, $x = 1$, and $x = 2$.

(a) Sketch the region $\mathcal{R}$. Label all relevant points and curves.



(b) Find the volume of the solid obtained by revolving $\mathcal{R}$ around the x-axis.

$$\text{Volume} = \pi \int_1^2 \left(\frac{1}{x^2}\right)^2 dx$$

$$= \pi \int_1^2 x^{-4}\, dx$$

$$= \pi \left(-\frac{1}{3} x^{-3}\right)\Big]_1^2$$

$$= \frac{\pi}{3}\left[-\frac{1}{8} + 1\right]$$

$$= \boxed{\frac{7\pi}{24}}$$

(c) Give an **expression**, in terms of a definite integral, for the solid obtained by revolving $\mathcal{R}$ around the line $y = -2$. You **do not** need to evaluate this integral!

$$\pi \int_1^2 \left(\left(2 + \frac{1}{x^2}\right)^2 - 2^2\right) dx$$

(d) Give an **expression**, in terms of a definite integral, for the solid obtained by revolving $\mathcal{R}$ around the line $x = -1$. You **do not** need to evaluate this integral!

WASHERS: $\pi \int_0^{1/4} \left(3^2 - 2^2\right) dx + \pi \int_{1/4}^{1} \left(\left(1 + \frac{1}{\sqrt{y}}\right)^2 - 2^2\right) dx$

$y = 1/x^2,\ x > 0 \Updownarrow x = 1/\sqrt{y}$

SHELLS: $2\pi \int_1^2 (1+x)\frac{1}{x^2}\, dx$