

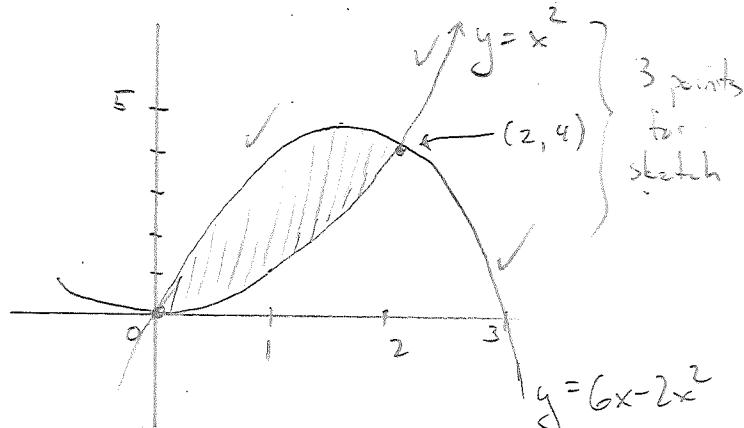
Name: SOLUTIONS

A#:

1. Let  $\mathcal{R}$  be the region bounded between the curves  $y = x^2$  and  $y = 6x - 2x^2$ .  $\rightarrow = 2x(3-x)$

(a) Sketch the region  $\mathcal{R}$ . Label all relevant points and curves.

$$\left. \begin{array}{l} x^2 = 6x - 2x^2 \\ \Leftrightarrow 3x^2 - 6x = 0 \\ \Leftrightarrow 3x(x-2) = 0 \\ \Leftrightarrow x=0 \text{ or } x=2 \\ \therefore \text{INT. PTS. ARE } (x,y) = (0,0) \text{ and } (x,y) = (2,4) \end{array} \right\} 3 \text{ pts}$$



(b) Find the volume of the solid obtained by revolving  $\mathcal{R}$  around the  $y$ -axis.

$$\begin{aligned} \text{Volume} &= 2\pi \int_0^2 x \underbrace{(6x - 2x^2 - x^2)}_{0 \leq x \leq 2} dx \\ &= 2\pi \int_0^2 (6x^2 - 3x^3) dx \\ &= 2\pi \left[ 2x^3 - \frac{3}{4}x^4 \right]_{x=0}^{x=2} \\ &= 2\pi (16 - 12 - 0) \\ &= \boxed{8\pi} \end{aligned} \quad \left. \begin{array}{l} \text{6 points} \\ \text{12} \end{array} \right\}$$

2. Give an expression, in terms of a definite integral, for the solid obtained by revolving the region  $\mathcal{R}$  in Question #1 around the given axis. Do not evaluate these integrals!

(a) The  $x$ -axis.

$$\pi \int_0^2 ((6x - 2x^2)^2 - (x^2)^2) dx$$

(b) The line  $x = 3$

$$2\pi \int_0^2 (3-x)(6x - 2x^2 - x^2) dx$$

(c) The line  $y = 8$

$$\pi \int_0^2 ((8-x^2)^2 - (8-6x+2x^2)^2) dx$$

3. Find the average value of the function  $f(t) = \frac{t}{\sqrt{9+t^2}}$  over the interval  $[0, 4]$ .

$$\begin{aligned} \text{Avg Value} &= \frac{1}{4-0} \int_0^4 \frac{t}{\sqrt{9+t^2}} dt \\ &= \frac{1}{4} \sqrt{9+t^2} \Big|_0^4 \\ &= \frac{1}{4} [\sqrt{25} - \sqrt{9}] \\ &= \boxed{\frac{1}{2}} \end{aligned}$$

4. Integrate the following:

$$(a) \int \sin^{-1} x dx \quad \begin{array}{l} \text{Let } u = \sin^{-1} x \\ du = \frac{1}{\sqrt{1-x^2}} dx \\ dv = dx \end{array} \Rightarrow \begin{array}{l} du = \frac{1}{\sqrt{1-x^2}} dx \\ v = x \end{array}$$

$$\begin{aligned} \text{Then } \int \sin^{-1} x dx &= x \sin^{-1} x - \int \frac{x}{\sqrt{1-x^2}} dx \\ &= \boxed{x \sin^{-1} x + \sqrt{1-x^2} + C} \end{aligned}$$

$$(b) \int_0^{\pi/4} x \cos 2x dx \quad \begin{array}{l} \text{Let } u = x \\ du = dx \\ dv = \cos 2x dx \\ v = \frac{1}{2} \sin 2x \end{array}$$

$$\begin{aligned} \text{Then } \int_0^{\pi/4} x \cos 2x dx &= \left[ x \cdot \frac{1}{2} \sin 2x \right]_0^{\pi/4} - \frac{1}{2} \int_0^{\pi/4} \sin 2x dx \\ &= \left( \frac{\pi}{4} \cdot \frac{1}{2} \cdot 1 - 0 \right) + \frac{1}{2} \cdot \frac{1}{2} \cos 2x \Big|_0^{\pi/4} \\ &= \boxed{-\frac{\pi}{8} - \frac{1}{4}} \end{aligned}$$

$$(c) \int \frac{\ln x}{\sqrt{x}} dx \quad \begin{array}{l} \text{Let } u = \ln x \\ du = \frac{1}{x} dx \\ dv = \frac{1}{\sqrt{x}} dx \\ v = 2\sqrt{x} \end{array}$$

$$\begin{aligned} \text{Then } \int \frac{\ln x}{\sqrt{x}} dx &= 2\sqrt{x} \ln x - \int 2\sqrt{x} \cdot \frac{1}{x} dx \\ &= 2\sqrt{x} \ln x - 2 \int \frac{1}{\sqrt{x}} dx \\ &= \boxed{2\sqrt{x} \ln x - 4\sqrt{x} + C} \end{aligned}$$