

Name: SOLUTIONS

A#:

$$1. \int \frac{x^3 + 6x^2 - x + 10}{(x^2 - 1)(x^2 + 2x + 5)} dx = \int \frac{x^3 + 6x^2 - x + 10}{(x-1)(x+1)(x^2 + 2x + 5)} dx$$

$$\text{Let } \frac{x^3 + 6x^2 - x + 10}{(x-1)(x+1)(x^2 + 2x + 5)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{Cx+D}{x^2 + 2x + 5}$$

$$\text{Then } x^3 + 6x^2 - x + 10 = A(x+1)(x^2 + 2x + 5) + B(x-1)(x^2 + 2x + 5) + (Cx+D)(x-1)(x+1)$$

$$\text{Set } x=1 \text{ to get } 16 = 16A \Rightarrow A = 1$$

$$\text{Set } x=-1 \text{ to get } 16 = -8B \Rightarrow B = -2$$

$$\text{Set } x=0 \text{ to get } 10 = 5A - 5B - D \Rightarrow D = 5$$

$$\text{Compare coeff. of } x^3 \text{ to get } 1 = A + B + C \Rightarrow C = 2$$

$$\text{Thus } \int \frac{x^3 + 6x^2 - x + 10}{(x^2 - 1)(x^2 + 2x + 5)} dx = \int \left(\frac{1}{x-1} - \frac{2}{x+1} + \frac{2x+5}{x^2 + 2x + 5} \right) dx \\ = \ln|x-1| - 2\ln|x+1| + \int \frac{2x+5}{x^2 + 2x + 5} dx$$

$$\begin{aligned} \text{Now: } \int \frac{2x+5}{x^2 + 2x + 5} dx &= \int \frac{2x+5}{(x+1)^2 + 4} dx = \int \frac{2u+3}{u^2 + 4} du \quad [u = x+1] \\ &= \int \frac{2u}{u^2 + 4} du + 3 \int \frac{du}{u^2 + 4} \\ &= \ln(u^2 + 4) + \frac{3}{2} \tan^{-1}\left(\frac{u}{2}\right) + C \\ &= \ln(x^2 + 2x + 5) + \frac{3}{2} \tan^{-1}\left(\frac{x+1}{2}\right) + C \end{aligned}$$

Altogether:

$$\int \frac{x^3 + 6x^2 - x + 10}{(x^2 - 1)(x^2 + 2x + 5)} dx = \ln|x-1| - 2\ln|x+1| + \ln(x^2 + 2x + 5) + \frac{3}{2} \tan^{-1}\left(\frac{x+1}{2}\right) + C$$

$$2. \int_0^2 \frac{2t}{(t-3)^2} dt \quad \text{Let } u = t-3, \text{ so } du = dt \text{ and } t = u+3$$

$$\begin{aligned} &= \int_{-3}^{-1} \frac{2(u+3)}{u^2} du \\ &= \left[\frac{2}{u} + \frac{6}{u^2} \right]_{-3}^{-1} \\ &= (2\ln|1| + 6) - (2\ln|3| + 6) \\ &= \boxed{4 - 2\ln 3} \end{aligned}$$

OR Partial Fractions ... $\frac{2t}{(t-3)^2} = \frac{A}{t-3} + \frac{B}{(t-3)^2}$

$$\Rightarrow A=2, B=6$$

$$\begin{aligned} \Rightarrow \int \frac{2t}{(t-3)^2} dt &= \int \left(\frac{2}{t-3} + \frac{6}{(t-3)^2} \right) dt \\ &= 2\ln|t-3| - \frac{6}{t-3} + C \end{aligned}$$

$$3. \int \cos \sqrt{x} dx$$

[Then evaluate between $t=0$ and $t=2$]

Substitute $u=\sqrt{x}$, so $du = \frac{1}{2\sqrt{x}} dx \Rightarrow dx = 2\sqrt{x} du = 2u du$

$$\text{Then } \int \cos \sqrt{x} dx = \int \cos(u) \cdot 2u du = 2 \int u \cos u du$$

But $\int u \cos u du = u \sin u - \int \sin u du$
 $= u \sin u + \cos u + C_1$

PARTS
 $\begin{cases} f=u & df=du \\ dg=\cos u du & g=\sin u \end{cases}$

$$\text{So } \int \cos \sqrt{x} dx = 2u \sin u + 2 \cos u + C$$

$$= \boxed{2\sqrt{x} \sin \sqrt{x} + 2 \cos \sqrt{x} + C}$$