**Math 1210: Quiz #1** **Winter 2017**

| | | |
|---|---|---|
| **Name:** SOLUTION | **A#:** | **Section:** A,B |

[4] 1. If $f(x) = \dfrac{2x}{x^2 - 2x + 4}$ and $g(x) = x^2 + 1$ then (do not simplify)

$f(g(x)) =$ $\dfrac{2(x^2+1)}{(x^2+1)^2 - 2(x^2+1) + 4}$

$g(f(x)) =$ $\left(\dfrac{2x}{x^2 - 2x + 4}\right)^2 + 1$

[4] 2. Circle equations of lines. (negative points for all wrong circles)

| | | | |
|---|---|---|---|
| $3x + 2y + 7 = 0$ | $y = 3x + \cos x$ | $y = 3x + \cos(y)$ | $x = \ln(7)$ |
| $5x + (\sin 1)y = 8$ | $(y - e^3) = 5(x - \cos 3)$ | $y - 1 = e^y(x - 3)$ | $xy = 0$ |

[4] 3. State the slope of the line, the $x$-intercept, the $y$-intercept, and the slope of any perpendicular line to the line $L$ given by the equation $2x + 5y + 3 = 0$.

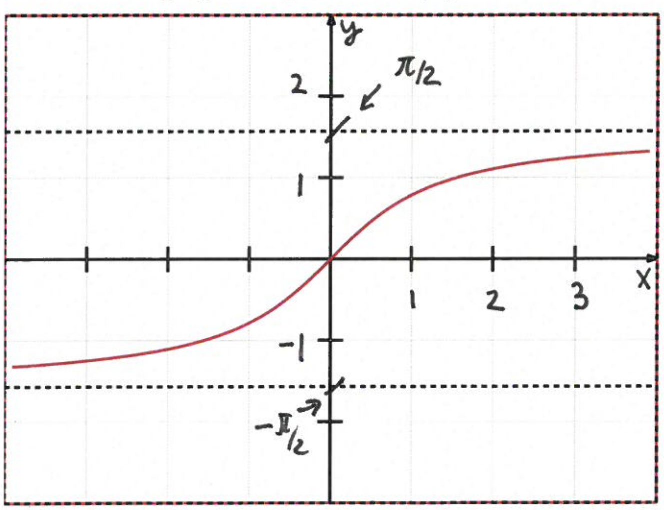
The slope of $L$ is $-\frac{2}{5}$

The $x-$ intercept of $L$ is $-\frac{3}{2}$

The $y-$ intercept of $L$ is $-\frac{3}{5}$

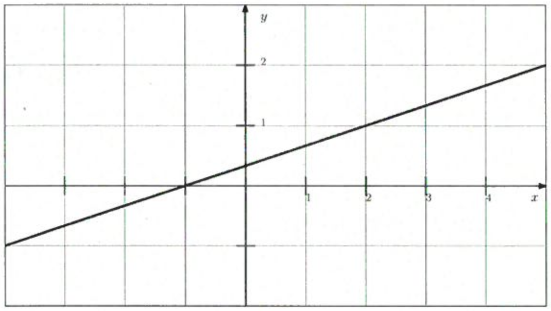
The slope of any line perpendicular to $L$ is $\frac{5}{2}$

[2] 4. Sketch the graph of $y = \tan^{-1}(x)$ below.




v1.AB

[2] 5. Find the equation of the line $L$ shown on the graph below.




equation of $L$ : $\boxed{y-1=\frac{1}{3}(x-2)}$ or $\boxed{y=\frac{1}{3}x+\frac{1}{3}}$

[4] 6. Find the equation of the secant line $L$ to the curve $y = x^3 - 2x - 1$ on the interval $[0, 2]$. *(Recall that a secant line to the curve $y = f(x)$ on the interval $[a, b]$ is the line passing through points $(a, f(a)), (b, f(b))$ and usually has no relationship to the $\sec x$ function.)*

Points : $(0,-1)$, $(2,3)$

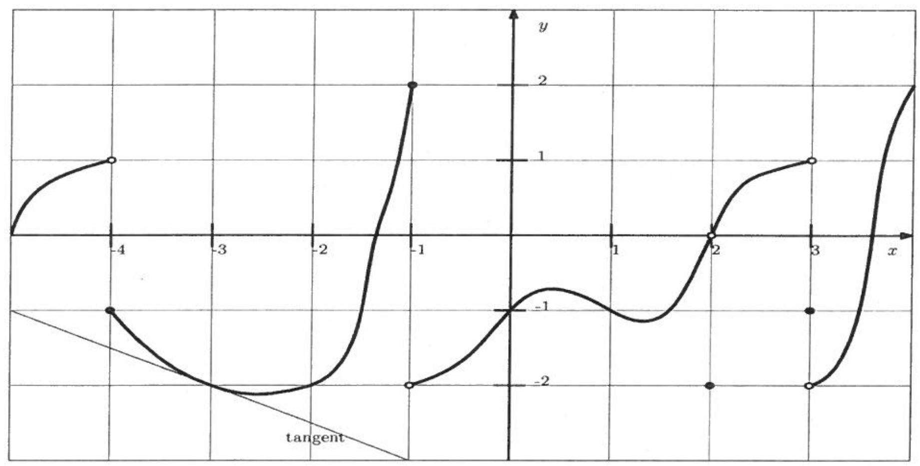
Slope $= \frac{3-(-1)}{2-0} = \frac{4}{2} = 2$

$L$: $\boxed{y+1 = 2(x-0)}$ or $\boxed{y = 2x-1}$

**Math 1210: Quiz #2** **Fall 2017**

| Name: SOLUTION | A#: | Section: A, B |
|---|---|---|

[9] 1. Let $f$ be a function whose graph of $y = f(x)$ is given below. Compute the following quantities or state that they do not exist.




(a) $f(3) =$ $-1$

(b) $\lim_{x\to 3} f(x)$ d.n.e.

(c) $\lim_{x\to 2}(x^2 + f(x))$ $= 4$

(d) $\lim_{x\to 1^-} f(x)$ $=$ $-1$

(e) $\lim_{x\to -1^+} f(x)$ $= -2$

(f) $\lim_{x\to -4^-} e^x f(x)$ $e^{-4}$

(g) The average rate of change of $f(x)$ over the interval $[-3, -1]$ $= \frac{4}{2} = \boxed{2}$

(h) The instantaneous rate of change of $f(x)$ when $x = -3$ $-\frac{1}{2}$

(i) The equation of the secant line over the interval $[-3, -1]$ $y + 2 = 2(x+3)$

or $y = 2x + 4$

v2.AB

[3] 2. Let $f(x) = \begin{cases} x^2 - 1, & \text{if } x < 2 \\ e^{x-2}, & \text{if } x \geq 2 \end{cases}$. Then

(a) $\lim_{x \to 2^-} f(x) =$ $2^2 - 1 = \boxed{3}$

(b) $\lim_{x \to 1^+} f(x) =$ $1^2 - 1 = \boxed{0}$

(and $\underline{\text{not}}$ $e^{1-2} = e^{-1}$!)

(c) The average rate of change of $f$ over the interval $[2, 4]$ is $\frac{e^2 - 1}{2}$

[8] 3. Compute the limit or state that it does not exist.

(a) $\lim_{x \to -2} \frac{x+2}{\sqrt{x^2+x+2} - 2} = \lim_{x \to -2} \frac{(x+2)(\sqrt{x^2+x+2}+2)}{(\sqrt{x^2+x+2}+2)(\sqrt{x^2+x+2}-2)}$

can skip these

$= \lim_{x \to -2} \frac{(x+2)(\sqrt{x^2+x+2}+2)}{(x^2+x+2) - 4} = \lim_{x \to -2} \frac{(x+2)(\sqrt{x^2+x+2}+2)}{x^2+x-2}$

$= \lim_{x \to -2} \frac{(x+2)(\sqrt{x^2+x+2}+2)}{(x+2)(x-1)} = \lim_{x \to -2} \frac{\sqrt{x^2+x+2}+2}{x-1} = \frac{\sqrt{(-2)^2-2+2}+2}{-2-1}$

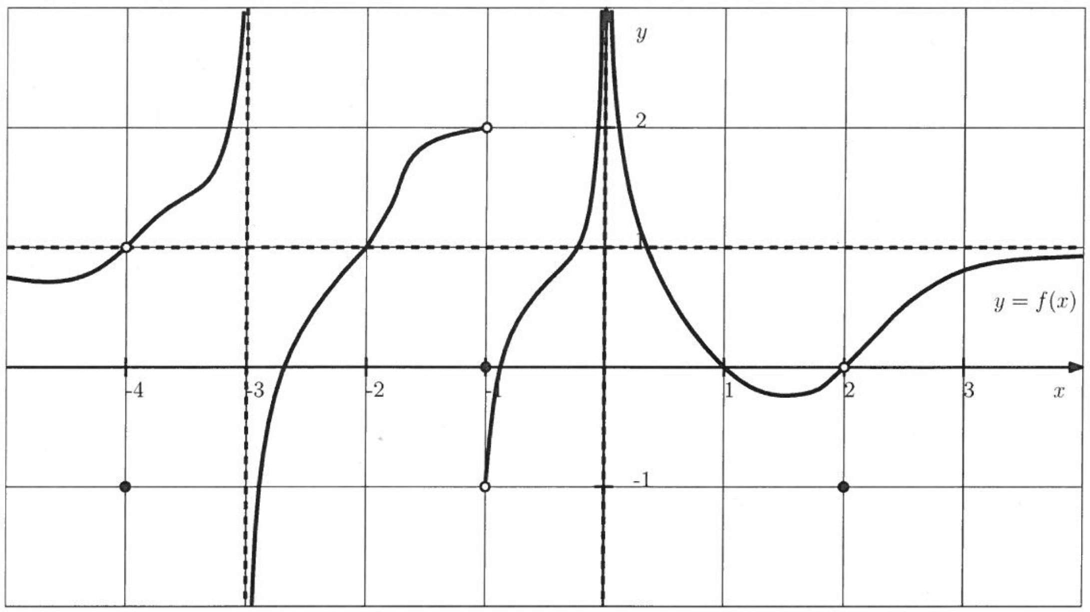
$= \boxed{-\frac{4}{3}}$

(b) $\lim_{x \to 3^-} \frac{|x-3|}{x^2-x-6} = \lim_{x \to 3^-} \frac{-(x-3)}{(x-3)(x+2)} = \lim_{x \to 3^-} \frac{-1}{x+2} = \frac{-1}{3+2} = \boxed{-\frac{1}{5}}$

**Math 1210: Quiz #3** **Fall 2017**

| Name: SOLUTION | A#: | Section: A, B |
|---|---|---|

[6] 1. Let $f$ be a function whose graph of $y = f(x)$ is given below.




Then

(a) $\lim_{z \to -4^-} f(z) =$ 1

(b) $\lim_{s \to -3^+} f(s) =$ $-\infty$

(c) $\lim_{z \to -1^-} f(z) =$ 2

(d) List all numbers $a$ for which $\lim_{s \to a} f(s)$ does not exist: $-3, -1, 0$

(e) List all horizontal asymptotes: $y = 1$

(f) List all vertical asymptotes: $x = -3,\ x = 0$

[2] 2. List all vertical asymptotes of $y = \dfrac{(x+2)^3(x-3)^2 \ln|x|}{(x+3)^2(x+2)^2(x-3)^3}$: $x = -3,\ x = 0,\ x = 3$

v3.AB

[4] 3. Find all horizontal asymptotes of $y = \dfrac{e^{2x} - 2e^{-3x}}{3e^{2x} + 5e^{-3x}}$.

$$\lim_{x \to \infty} \frac{e^{2x} - 2e^{-3x}}{3e^{2x} + 5e^{-3x}} = \lim_{x \to \infty} \frac{1 - 2e^{-5x}}{3 + 5e^{-5x}} = \frac{1-0}{3+0} = \boxed{\frac{1}{3}}$$

$$\lim_{x \to -\infty} \frac{e^{2x} - 2e^{-3x}}{3e^{2x} + 5e^{-3x}} = \lim_{x \to -\infty} \frac{e^{5x} - 2}{3e^{5x} + 5} = \frac{0-2}{0+5} = \boxed{-\frac{2}{5}}$$

Horizontal Asymptotes: $\boxed{y = \frac{1}{3}}$ and $\boxed{y = -\frac{2}{5}}$

[8] 4. Compute the limits or show that they do not exist.

(a) $\displaystyle\lim_{t \to \infty} \frac{t^2 + \sin t}{3t^2 - 2\ln(t)} = \lim_{t \to \infty} \frac{1 + \frac{\sin t}{t^2}}{3 - \frac{2\ln(t)}{t^2}} = \frac{1+0}{3-0} = \boxed{\frac{1}{3}}$

(b) $\displaystyle\lim_{x \to \infty} \left(\sqrt{x^2 - 2x + 5} - x\right) = \lim_{x \to \infty} \frac{(\sqrt{x^2-2x+5} - x)(\sqrt{x^2-2x+5} + x)}{\sqrt{x^2-2x+5} + x}$

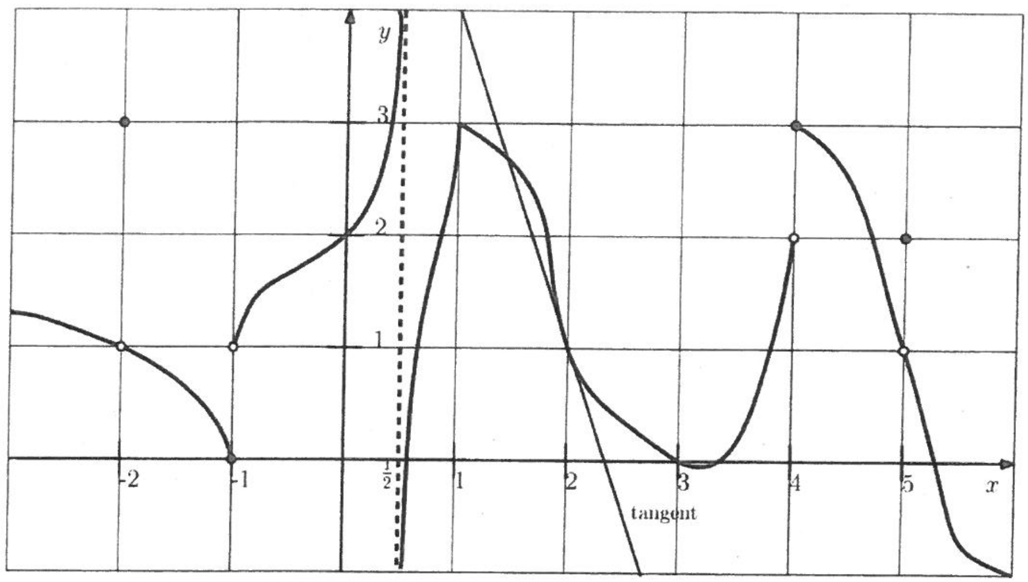
$$= \lim_{x \to \infty} \frac{(x^2-2x+5) - x^2}{\sqrt{x^2-2x+5} + x} = \lim_{x \to \infty} \frac{-2x+5}{\sqrt{x^2-2x+5} + x} = \lim_{x \to \infty} \frac{-2 + \frac{5}{x}}{\sqrt{1 - \frac{2}{x} + \frac{5}{x^2}} + 1}$$

$$= \frac{-2+0}{\sqrt{1+0+0} + 1} = \boxed{-1}$$

**Math 1210: Quiz #4** **Fall 2017**

| Name: SOLUTION | A#: | Section: A, B |
|---|---|---|

[8] 1. Let $f$ be a function whose graph of $y = f(x)$ is given below.




Fill in the following.

(a) List all $x$ where $f$ is not continuous: $-2, -1, \frac{1}{2}, 4, 5$

(b) List all $x$ where $f$ is continuous, but not differentiable: $1$

(c) List all $x$ where $f$ is right-continuous, but not continuous: $4$

(d) $\lim_{x\to 5}(f(x)+1)^2 =$ $(1+1)^2 = \boxed{4}$

(e) ~~lim f(e) =~~ $\lim_{x\to 0} f(e^x) = f(1) = \boxed{3}$

(f) $f'(2) =$ $-3$

(g) If $g(x) = x^2 f(x)$, then $g'(2) =$ $\left(2x f(x) + x^2 f'(x)\right)\Big|_{x=2} = 2\cdot 2\cdot 1 + 2^2\cdot(-3) = \boxed{-8}$

(h) If $h(x) = f(2x)$, then $h'(1) =$ $2f'(2x)\Big|_{x=1} = 2f'(2) = \boxed{-6}$

v4.AB

[4] 2. Find the equation of the tangent line to $y = x^2 + 1$ at $x = 2$.

$y' = 2x$, $y|_{x=2} = (2)^2 + 1 = 5$, $y'|_{x=2} = 2(2) = 4$

Line: $\boxed{y - 5 = 4(x-2)}$ or $\boxed{y = 4x - 3}$

[8] 3. Compute the derivative. **Do not simplify your answer.**

(a) $\frac{d}{dx}\left(\tan(x) + \frac{1}{x^2} + e^{2x} + \sin(4)\right) = \sec^2 x - 2x^{-3} + 2e^{2x} + 0$

$x^{-2}$

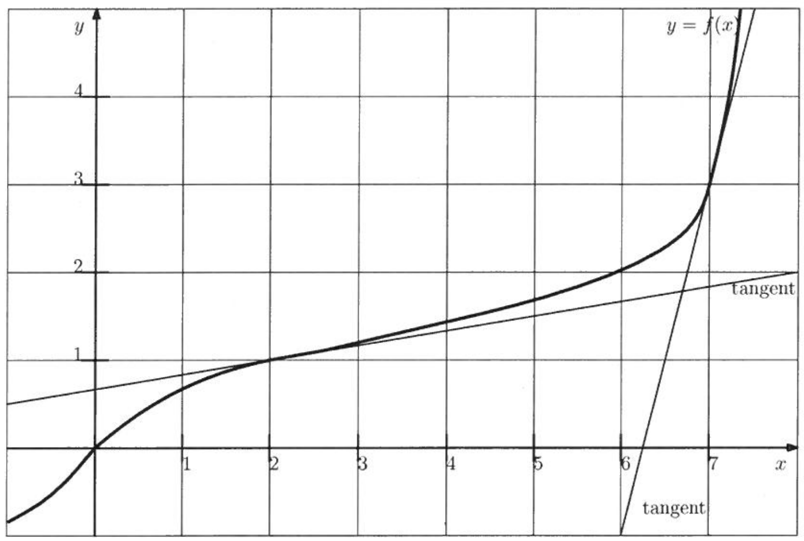
(b) $\frac{d}{dt}\left(\frac{\sin(t)}{e^t + 1}\right) = \frac{(\cos t)(e^t + 1) - (\sin t)e^t}{(e^t + 1)^2}$.

v4.AB

**Math 1210: Quiz #5** **Fall 2017**

| Name: SOLUTION | A#: | Section: |
|---|---|---|

[8] 1. Let $f$ be a function whose graph of $y = f(x)$ is given below and let $g = f^{-1}$ be its inverse function.




Fill in the following.

(a) $g(2) =$ 6

(b) $\lim_{h\to 0} \frac{f(2+h)-f(2)}{h} =$ $f'(2) = \boxed{\frac{1}{6}}$

(c) The instantaneous rate of change of $f(x)$ when $x = 7$ is 4

(d) $g'(3) =$ $\frac{1}{f'(7)} = \boxed{\frac{1}{4}}$

(e) If $h(x) = f(2x^2 - 1)$, then $h'(2) =$ $4x\, f'(2x^2-1)\big|_{x=2} = 4(2)f'(7) = \boxed{32}$

(f) If $k(x) = g(x)\ln(x)$, then $k'(3) =$ $\left(g'(x)\ln(x) + \frac{g(x)}{x}\right)\Big|_{x=3} = \boxed{\frac{1}{4}\ln(3) + \frac{7}{3}}$

(g) If $F(x) = \tan^{-1}(f(x))$, then $F'(2) =$ $\frac{f'(x)}{1+(f(x))^2}\Big|_{x=2} = \frac{f'(2)}{1+1^2} = \boxed{\frac{1}{12}}$

(h) Tangent line to the curve $y = 2f(x)$ at $x = 2$ is $\boxed{y - 2 = \frac{1}{3}(x-2)}$

$y|_{x=2} = 2f(2) = 2$

$y' = 2f'(x)$, $y'|_{x=2} = 2f'(2) = \frac{1}{3}$

v5.AB

[4] 2. Compute the derivative. **Do not simplify.**

3. $\frac{d}{dt}\left(\sec(e^t) + \tan^{-1}(4) + \sin^{-1}(2t) + \ln(t^4+1)\right)$

$$= \cancel{\sec}\ \sec(e^t)\tan(e^t)\,e^t + 0 + \frac{1}{\sqrt{1-(2t)^2}}2 + \frac{4t^3}{t^4+1}.$$

[8] 4. Consider the curve given by $xy^2 = 5 + x^2 + y$. Find the equation of the tangent line given to the curve at the point $(3, -2)$.

$$y^2 + 2xyy' = 2x + y'$$

$$y'(2xy-1) = 2x - y^2$$

$$y' = \frac{2x-y^2}{2xy-1}$$

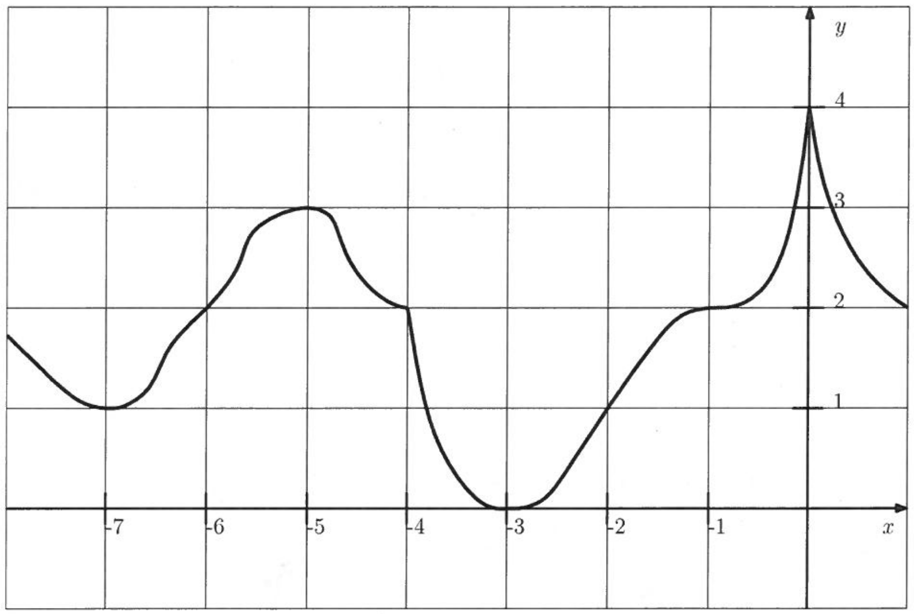
$$y'\Big|_{\substack{x=3\\y=-2}} = \frac{2(3)-(-2)^2}{2(3)(-2)-1} = \frac{2}{-13} = \boxed{-\frac{2}{13}}$$

Tangent: $\boxed{y+2 = -\frac{2}{13}(x-3)}$

# Math 1210: Quiz #6 — Fall 2017

| Name: SOLUTION | A#: | Section: |
|---|---|---|

[8] 1. Let $f$ be a function whose graph of $y = f(x)$ is given below.




Fill in the following.

(a) The critical ~~values of~~ points for $f$ are: $-7, -5, -4, -3, -1, 0$

(b) $f$ has local minima at: $-7, -3$

(c) $f$ has local maxima at: $-5, 0$

(d) On the following intervals we have $f'(x) > 0$: $(-7, -5), (-3, -1), (-1, 0)$

(e) The global maximum of $f$ on $(-8, 1)$ is $4$

(f) The global minimum of $f$ on $(-8, 1)$ is $0$

(g) The global maximum of $f$ on $[-7, -6]$ is $2$

(h) The global minimum of $f$ on $[-2, 0]$ is $1$

v6.AB

[3] 2. List and classify critical points of $f$, if its **derivative** is given by

$$f'(x) = \frac{(x-2)^3x^2}{\sqrt[3]{x+2}}$$

$$\begin{array}{ccccccc} f' & + & & - & & - & & + \\ & & -2 & & 0 & & 2 & \end{array}$$

Critical points : $-2$ (local max.), $0$ (not a local extremum), $2$ (loc. min)

[4] 3. Let $x, y$ be functions of $t$ related by $4x^2y^2 = x^4 + y^4$. Compute $\frac{dy}{dt}$ in terms of $x, y, \frac{dx}{dt}$.

$$8xy^2\frac{dx}{dt} + 8x^2y\frac{dy}{dt} = 4x^3\frac{dx}{dt} + 4y^3\frac{dy}{dt}$$

$$\frac{dy}{dt}(8x^2y - 4y^3) = \frac{dx}{dt}(4x^3 - 8xy^2)$$

$$\frac{dy}{dt} = \boxed{\frac{dx}{dt}\cdot\frac{4x^3-8xy^2}{8x^2y-4y^3}} = \frac{dx}{dt}\;\frac{x^3-2xy^2}{2x^2y-y^3}$$

[5] 4. Find the global (absolute) maximum and the global minimum of $f(x) = x^3 + 6x^2$ on $[-5, -1]$.

$f'(x) = 3x^2 + 12x = 3x(x+4)$.

Critical points : $-4$, $0$ ← not in the interval

| $x$ | $f(x)$ | |
|---|---|---|
| $-5$ | $25$ | |
| $-4$ | $32$ | max |
| $-1$ | $5$ | min |

$f(-5) = (-5)^3 + 6(-5)^2 = -125 + 150 = 25$

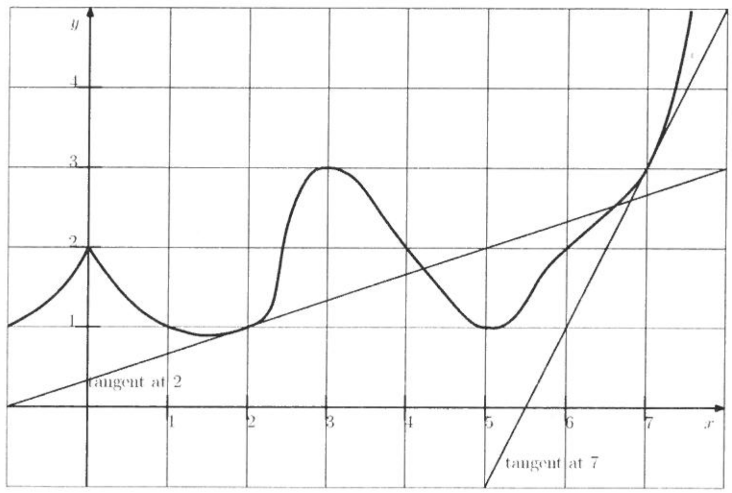
$f(-2) = (-4)^3 + 6(-4)^2 = -64 + 96 = 32$

$f(-1) = (-1)^3 + 6(-1)^2 = 5$.

**Math 1210: Quiz #8** **Fall 2017**

| Name: SOLUTION | A#: | Section: |
|---|---|---|

[7] 1. Let $f$ be a function whose graph of $y = f(x)$ is given below. Let $L_2$ be the linearisation (linear approximation) of $f$ centred at 2 and let $L_7$ be the linearisation of $f$ centred at 7.




Fill in the following.

(a) For the following values of $x$ we have $f'(x) = 0$: 3, 5

(b) The global maximum of $f(x)$ on the interval $(4, 7]$ is: 3

(c) The global maximum of $f(x)$ on the interval $(-1, 6)$ is: 3

(d) $L_2(x) =$ $1 + \frac{1}{3}(x-2)$

(e) The error in estimating $f(5) \approx L_7(5)$ is 2

(f) If $dy$ is the differential of $y = f(3+x^2)$ centred at 2, then $dy(dx) =$ $8\,dx$

and $dy(-1) =$ $-8$

dx is the variable

$y' = 2x f'(3+x^2)$

v8.AB

$y'|_{x=2} = 2 \cdot 2 \cdot f'(7) = 8$

2. Let $f(x) = e^x$.

[6] (a) Find the linearisation (linear approximation) $L(x)$ of $f(x)$ centred at 0.

$f'(x) = e^x, \quad f'(0) = 1, \quad f(0) = 1$

$\boxed{L(x) = 1 + x}$

[2] (b) Use the linearisation above to estimate $e^{0.1}$.

$e^{0.1} \approx L(0.1) = 1.1$

[5] (c) Is $L(0.1)$ larger or smaller then $e^{0.1}$? Justify your answer.

$f''(x) = e^x$, this is positive on $[0, 0.1]$ so

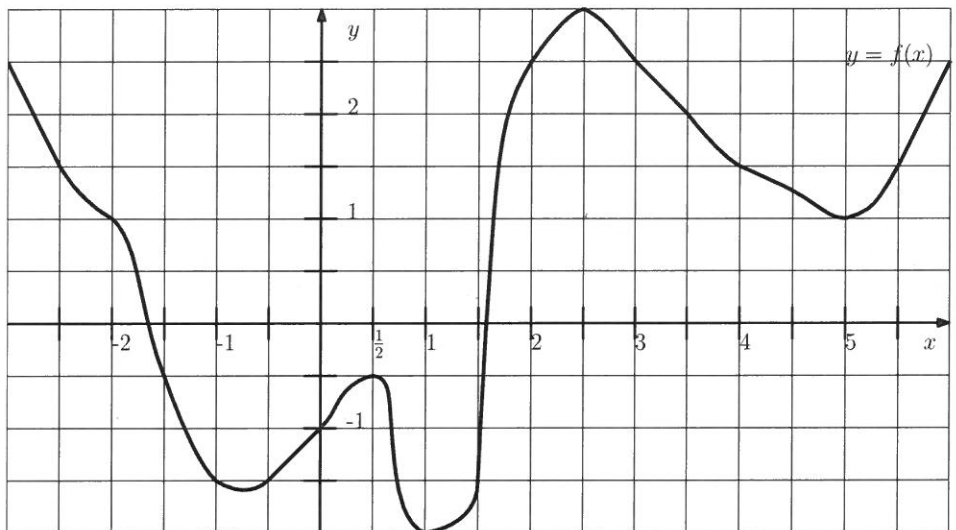
$L(0.1) < e^{0.1}$.

v8.AB

## Math 1210: Quiz #9 Fall 2017

| Name: SOLUTION | A#: | Section: |
|---|---|---|

[7] 1. Let $f$ be a function whose graph of $y = f(x)$ is given below. Let $L_N$ and $R_N$ denote the left-endpoint, respectively right end-point, Riemann sums with $N$ subintervals of equal size. Fill in the following.




(a) Estimate the area under the curve over [2, 5] by $L_2$: $\frac{3}{2}\left(\frac{5}{2}+2\right)=\boxed{\frac{27}{4}}$

(b) Estimate the area under the curve over [2,4] by $R_4$: $\frac{2}{4}\left(3+\frac{5}{2}+2+\frac{3}{2}\right)=\boxed{\frac{18}{4}}=\boxed{\frac{9}{2}}$

(c) Estimate $\int_{-1}^{2} f(x)dx$ by $L_2$: $\frac{3}{2}\left(-\frac{3}{2}-\frac{1}{2}\right)=\boxed{-3}$

(d) Estimate $\int_{0}^{3}\left(f\left(\frac{x}{2}\right)+1\right)^2 dx$ by $R_3$: $1\cdot\left(\left(f\left(\frac{1}{2}\right)+1\right)^2+\left(f\left(\frac{2}{2}\right)+1\right)^2+\left(f\left(\frac{3}{2}\right)+1\right)^2\right)=\frac{1}{4}+1+\frac{1}{4}=\boxed{\frac{3}{2}}$

(e) If $F(x)=\int_0^x f(t)dt$, then $F'(3) =$ $f(3)=\boxed{\frac{5}{2}}$

(f) If $G(x)=\int_{-1}^{x^2} t^2 f(t+1)dt$, then $G'(x) =$ $2x\left((x^2)^2 f(x^2+1)\right) = 2x^5 f(x^2+1)$

and $G'(2) =$ $2(2)^5 f(5)=\boxed{64}$

v9.AB

[4] 2. Solve the initial value problem $\frac{dy}{dx} = 3e^{2x}$, $y(0) = 4$.

$y = \int 3e^{2x}\,dx = \frac{3}{2}e^{2x} + C$, $4 = y(0) = \frac{3}{2}e^{0} + C \Rightarrow C = 4 - \frac{3}{2} = \frac{5}{2}$

$\therefore \boxed{y = \frac{3}{2}e^{2x} + \frac{5}{2}}$

3. Compute the integral.

[5] (a) $\int \left(\sin(2x) + \frac{2}{1+x^2} + \sec^2(3x) + e^3 + x^{-3}\right) dx$

$= -\frac{1}{2}\cos(2x) + 2\tan^{-1}(x) + \frac{1}{3}\tan(3x) + e^3 x + \frac{x^{-2}}{-2} + C$

[4] (b) $\int_1^2 \frac{1+t^2}{t} dt = \int_1^2 \left(\frac{1}{t} + t\right) dt = \left(\ln|t| + \frac{t^2}{2}\right)\Big|_1^2 = \left(\ln 2 + \frac{2^2}{2}\right) - \left(\ln 1 + \frac{1^2}{2}\right)$

$= \boxed{(\ln 2) + \frac{3}{2}}$

| Name: Solution | A#: | Section: |
|---|---|---|

[5] 1. $\int (2x+1)(x^2+x+2)^{17}dx$

$$\begin{cases} \text{let } u = x^2+x+2 \\ du = (2x+1)\,dx \end{cases}$$

$$= \int u^{17}\,du$$

$$= \frac{u^{18}}{18} + C$$

$$= \frac{1}{18}(x^2+x+2)^{18} + C$$

[5] 2. $\int \sec^2(x)e^{\tan(x)}dx$

let $u = \tan x$

$du = \sec^2 x \, dx$

$$= \int e^u\,du$$

$$= e^u + C$$

$$= e^{\tan x} + C$$

v10.AB

[5] 3. $\int_1^e \frac{\ln(x)}{x} dx$

$$\begin{cases} \text{let } u = \ln x \\ du = \frac{1}{x} dx \end{cases} \qquad \begin{cases} x = 1 \rightarrow u = \ln 1 = 0 \\ x = e \rightarrow u = \ln e = 1 \end{cases}$$

$= \int_0^1 u \, du$

$= \left[ \frac{u^2}{2} \right]_0^1$

$= \frac{1}{2} \left[ u^2 \right]_0^1$

$= \frac{1}{2} (1^2 - 0)$

$= \boxed{\frac{1}{2}}$

[5] 4. $\int_0^{\frac{\pi}{2}} \frac{\cos(x)}{\sin(x) + 1} dx$

$$\begin{cases} \text{let } u = \sin x + 1 \\ du = \cos x \, dx \end{cases} \qquad \begin{cases} x = 0 \rightarrow u = \sin(0) + 1 = 1 \\ x = \frac{\pi}{2} \rightarrow u = \sin \frac{\pi}{2} + 1 = 2 \end{cases}$$

$\int_1^2 \frac{1}{u} du$

$= \ln |u| \Big]_1^2$

$= \ln 2 - \ln 1$

$= \boxed{\ln 2}$