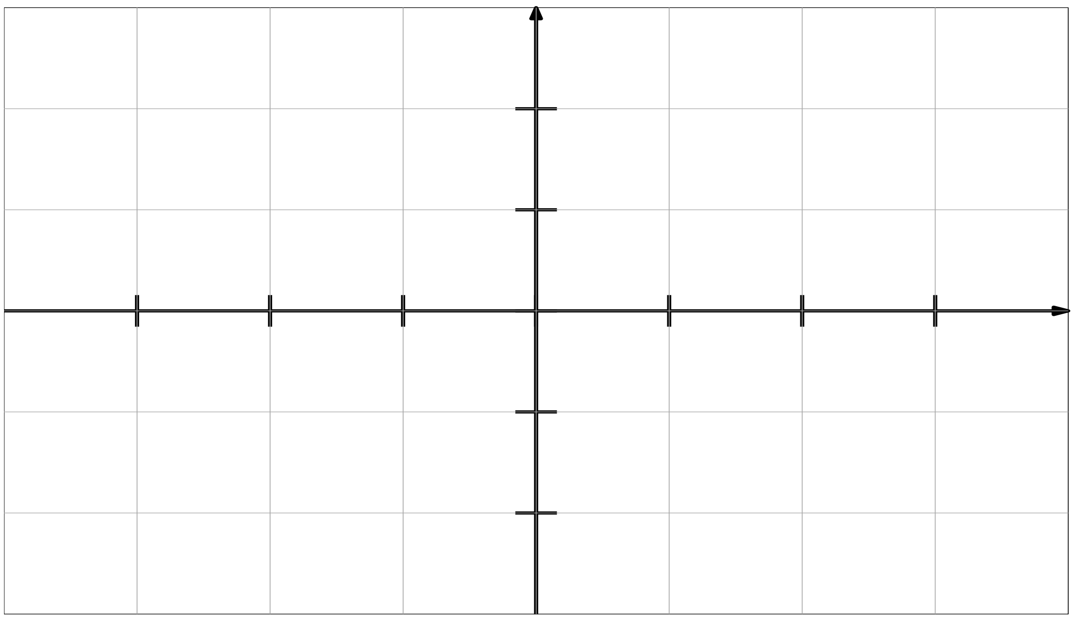
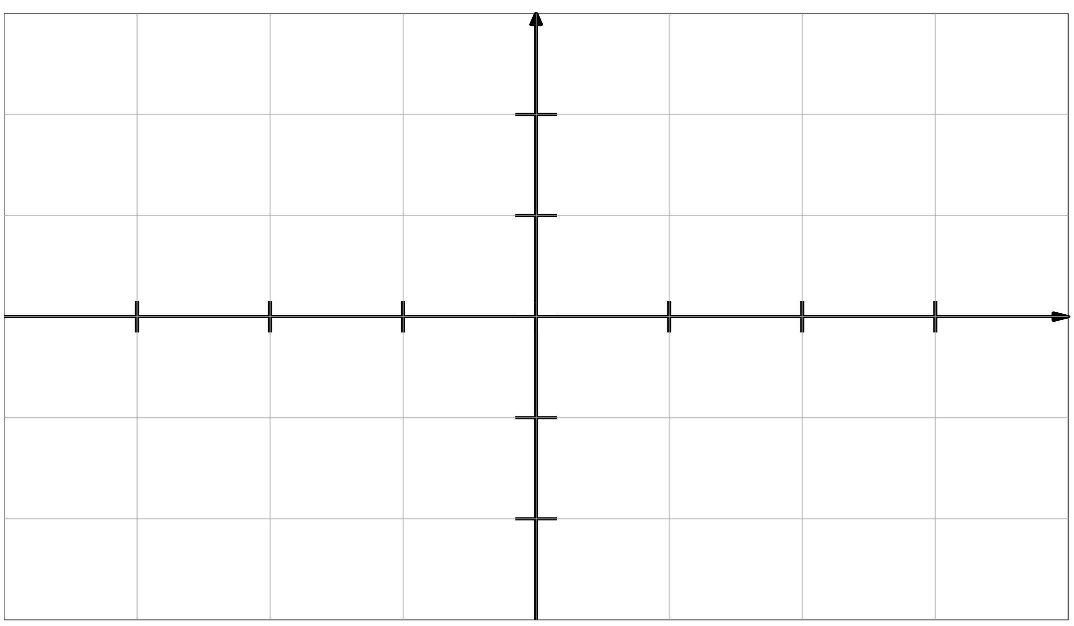
Math 1210: Worksheet #1 — Winter 2017

Name:	A#:	Section:

[4] 1. In the coordinate system below sketch the graphs of the functions $y = e^x$ and $y = \ln(x)$. Make sure that you clearly label the coordinate system and each curve.



[4] 2. In the coordinate system below sketch the graphs of the functions $y = \sin(x)$, $y = \cos(x)$. Make sure that you clearly label the coordinate system and each curve.



[4] 3. Find the equation of the secant line to the curve $y = e^x$ on the interval $[\ln(2), \ln(6)]$. Write the equation in the "slope, y-intercept form" and simplify as much as possible.

[**2**] 4. If $f(x) = \ln(x^2 + x + 1)$, $g(x) = e^x$ and $h(x) = x + 1$, then

$(f \circ g \circ h)(x) =$ ____

[**6**] 5. Perform the required task and **simplify**.

(a) Write as a sum of summands of the form x^a. $\dfrac{x^2 + \sqrt[5]{x}}{x^3} =$

(b) Put on common denominator. $\dfrac{1}{x-2} - \dfrac{2}{x} =$

(c) Expand. $(x^2 + x + 3)(x^2 - 2x + 3) =$

(d) Rationalize and cancel if possible. $\dfrac{x+2}{\sqrt{x^2 + x + 7} - 3} =$

(e) $\dfrac{(e^x)^{x-1} e^{x-1}}{e^{2-3x}} =$

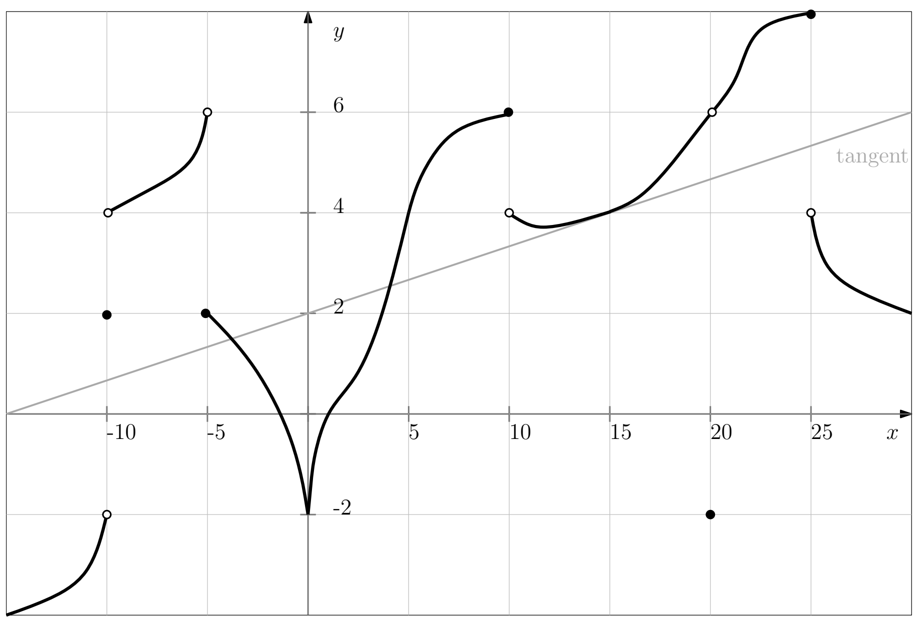
(f) $\ln(3e^x/5) + \ln(5e^x) - \ln(3e^{x^3}) =$

Math 1210: Worksheet #2 **Fall 2017**

Name:	**A#:**	**Section:**

[2] 1. State the formal (ε, δ) definition of $\lim_{x \to a} f(x) = L$. Part marks will be given for an informal definition.

[5] 2. Let f be a function whose graph of $y = f(x)$ is given below.



(a) $\lim_{x \to 15} \dfrac{f(x)}{x^2 - 10x} =$ ______________

(b) $\lim_{x \to 10^+} (2\ln(x) - 2f(x)) =$ ______________

(c) List all values of a in the interval $(-15, 30)$ for which limit $\lim_{x \to a} f(x)$ does not exist:

____

(d) The average rate of change of $f(x)$ over the interval $[-5, 10]$ is __________

(e) The instantaneous rate of change of $f(x)$ when $x = 15$ is __________

[3] 3. Let $f(x) = \frac{1}{x}$ and let $a \neq 0$. Find the instantaneous rate of change of $f(x)$ when $x = a$, as a function (call it g) of a (you are not allowed to use any theory of derivatives).

$g(a) = \lim$

4. Compute the limit or show that it does not exist.

[3] (a) $\displaystyle\lim_{x\to 2} \frac{\sqrt{x^2+5}-\sqrt{5x-1}}{x^2+3x-10}$

[3] (b) $\displaystyle\lim_{x\to 2^-} \Big(\ln(2-x)-\ln(6-x-x^2)\Big)$

[4] (c) $\displaystyle\lim_{x\to 2} \frac{|4-x^2|}{x^2-x-2}$

Math 1210: Worksheet #3 **Fall 2017**

Name:	**A#:**	**Section:**

[**2**] 1.

(a) List vertical asymptotes of

$$y = \frac{x^2+3x-10}{x^2-4} + \ln|x^2-x|.$$

(b) List horizontal asymptotes of $y = \tan^{-1}(x)$.

[**6**] 2. Find horizontal asymptotes of $y = \dfrac{1-3x}{\sqrt{x^2-x+3}}$

[12] 3. Compute the limits (as a number, as ∞ , as $-\infty$, or as 'does not exist'; whichever is most precise).

(a) $\displaystyle\lim_{x\to 1^+} \frac{x-2}{x^2+2x-3}$

(b) $\displaystyle\lim_{x\to 0} \frac{2\sin x}{x} + \lim_{t\to -\infty} \frac{\sin t}{2t} + \lim_{s\to \frac{\pi}{6}} \frac{\sin s}{s}$

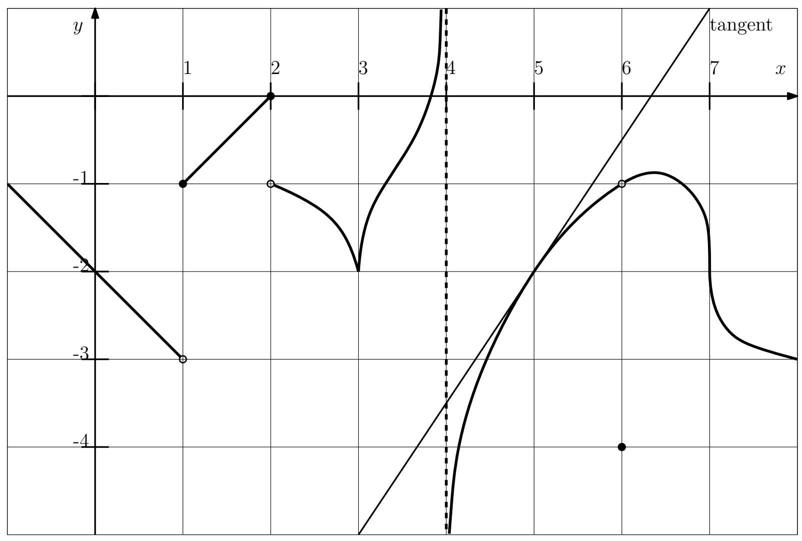
(c) $\displaystyle\lim_{x\to \infty} \left(e^x - \sqrt{e^{2x} - e^x + 1}\right)$

(d) $\displaystyle\lim_{x\to -\infty} \left(x^3 - 3x + 2e^{-x}\right)$

Math 1210: Worksheet #4 Fall 2017

Name:	A#:	Section:

[**8**] 1. Let f be a function whose graph of $y = f(x)$ is given below.



Fill in the following.

(a) List all x where f is not continuous: ____________________

(b) List all x where f is left-continuous, but f is not right-continuous: ____________________

(c) List all x where f is continuous, but not differentiable: ____________________

(d) $\lim_{t \to 4^+} e^{f(t)} =$ ____________________

(e) If $g(x) = \dfrac{f(2x+1)}{x^2+1}$, then $g'(x) =$ ____________________ and $g'(2) =$ ____________

(f) If $h(x) = e^x f(x)$, then $h'(x) =$ ____________________ and the equation of the tangent line to the curve $y = h(x)$ at $x = 5$ is ____________________

[**2**] 2. Find a function f and a number a such that

$$f'(a) = \lim_{h \to 0} \frac{(3+h)^3 + \tan^{-1}(3(3+h)+2) - 27 - \tan^{-1}(11)}{h}$$

$a =$ ________________

$f(x) =$ ____________________

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[**6**] 3. Find values of a and b such that the function

$$f(x) = \begin{cases} \tan^{-1}\left(\frac{1}{1-x}\right) & , \text{ for } x < 1 \\ a & , \text{ for } x = 1 \\ b\sin^{-1}\left(\frac{x-1}{x^2-1}\right) & , \text{ for } x > 1 \end{cases}$$

is continuous everywhere.

[**4**] 4. Compute the derivative. **Do not simplify.**

$$\frac{d}{dt}\left(\frac{\sqrt[3]{t}\cos(t)}{1+\sec(2t+1)}\right)$$

Math 1210: Worksheet #5 **Fall 2017**

Name:	**A#:**	**Section:**

[12] 1. Compute the derivative. **Do not simplify.**

(a) $\dfrac{d}{du}\left(\sec(e^{\sqrt{u}})\right)$

(b) $\dfrac{d}{dx}\left(x\ln(e^x+1)\tan^{-1}(x^2)\right)$

(c) $\dfrac{d}{dx}\left((\sin^{-1}(x))^{\sin(x)}\right)$

(d) $\dfrac{d}{dt}\dfrac{\ln(t^4+1)}{t^4+1}$

[4] 2. Find the equation of the tangent line to the curve

$$x + y - 1 = x\cos(y)$$

at the point $(0, 1)$.

[4] 3. Find the equation of the tangent line to the curve

$$3(x^2 + y^2)^2 = 25(x^2 - y^2)$$

at the point $(2, 1)$. (the curve is called lemniscate)

Math 1210: Worksheet #6 **Fall 2017**

Name:	**A#:**	**Section:**

[**6**] 1. Consider a right-angled triangle with catheti (adjacent sides) x and y and hypothenuse (opposite side) z. When $x = 5$ and $y = 12$ we have that x is growing at the rate of $7cm/s$ and the angle θ between x and z is growing at the rate of $\frac{1}{4}rad/s$, (the right angle $\frac{\pi}{2}$ between x and y is fixed throughout the process). What is the rate of change of z at that point?

[**3**] 2. Let x and y be functions of t related by $x^2 + y^2 = xy + 7$. When $x = 3$ and $y = 2$ we have that $\frac{dx}{dt} = -2$. Find the value of $\frac{dy}{dt}$ at that point.

[3] 3. Let f be a function whose **derivative** is

$$f'(x) = \frac{(x+2)^4 \ln(x^2) e^x}{\sqrt[3]{x+9}}.$$

List the intervals where f is increasing and list as well as classify all critical values.

[8] 4. Find the global maximum and the global minimum of $f(x) = (x^2 - 2x)e^x$ on the interval $[0, 3]$. For **5 bonus marks** find the global maximum and minimum of f on $(-\infty, 0]$ or explain why they do not exist.

Math 1210: Worksheet #7 **Fall 2017**

Name:	**A#:**	**Section:**

[**8**] 1. Sketch the graph of $y = xe^{-x^2/2}$. You can use $y' = (1-x^2)e^{-x^2/2}$ and $y'' = x(x^2-3)e^{-x^2/2}$. Make sure that your graph clearly shows any x and y intercepts, horizontal asymptotes, vertical asymptotes, local extrema, intervals where the function is increasing and where it is decreasing, concavity, and inflection points.

[8] 2. Sketch the graph of $y = \frac{x}{1-x^2}$. You can use $y' = \frac{1+x^2}{(1-x^2)^2}$ and $y'' = \frac{2x(x^2+3)}{(1-x^2)^3}$. Make sure that your graph clearly shows any x and y intercepts, horizontal asymptotes, vertical asymptotes, local extrema, intervals where the function is increasing and where it is decreasing, concavity, and inflection points.

[4] 3. Consider a hypothetical function f defined everywhere with the property that f has a discontinuity at 3 and critical points at $-5, -3, 1, 7, 9$. Find the global maximum and global minimum, or state that they do not exists, of f on $I = [0, \infty)$ for the following cases.

(a)

$\lim_{x\to-\infty} f(x)$	$f(-5)$	$f(-3)$	$f(0)$	$f(1)$	$\lim_{x\to3^-} f(x)$	$f(3)$	$\lim_{x\to3^+} f(x)$	$f(7)$	$f(9)$	$\lim_{x\to\infty} f(x)$
∞	-42	15	7	0	-7	-5	9	10	4	3

Maximum ____________________ **Minimum** ____________________

(b)

$\lim_{x\to-\infty} f(x)$	$f(-5)$	$f(-3)$	$f(0)$	$f(1)$	$\lim_{x\to3^-} f(x)$	$f(3)$	$\lim_{x\to3^+} f(x)$	$f(7)$	$f(9)$	$\lim_{x\to\infty} f(x)$
$-\infty$	-42	5	5	7	5	9	9	10	14	∞

Maximum ____________________ **Minimum** ____________________

Math 1210: Worksheet #8 **Fall 2017**

Name:	**A#:**	**Section:**

[**5**] 1. Find the linearisation $L(x)$ of $f(x) = \tan^{-1}(x)$ centred at 1.

[**5**] 2. Estimate $\sqrt{23}$. Is your estimate larger or smaller then $\sqrt{23}$? Justify your answer.

[5] 3. Find the point(s) on the curve $y = \sqrt{x+1}$, $x \geq -1$, that is closest to the point $(0, 0)$. (Hint: minimize the square of the distance)

[5] 4. Find the point(s) (x, y) on the curve $y = -x^3 + 8x^2 - 10x$ with largest value of $P = xy$.

Math 1210: Worksheet #Fall Break **Fall 2017**

Name:	**A#:**	**Section:**

1. Compute the limit and explain why L'Hôpital's rule does not apply.

   (a) $\displaystyle\lim_{x\to\frac{\pi}{4}} \frac{\sin x}{x}$.

   (b) $\displaystyle\lim_{x\to\infty} \frac{\ln x + 3x + \cos x}{3\ln x + 3x + 3\cos x}$.

2. Compute the limit.

   (a) $\displaystyle\lim_{x\to\frac{\pi}{2}} (2x-\pi)\sec x$

   (b) $\displaystyle\lim_{x\to 1} \frac{x - \ln x - 1}{x^2 - 2x + 1}$

(c) $\lim_{x\to 0^+} (\sin x)^x$

(d) $\lim_{x\to 0} \dfrac{e^{2x} - 1 - 2x - 2x^2}{x\cos x - x}$

3. Prove that the functions $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$ differ by a constant. (Hint: compute the derivatives.)

4. Compute the indefinite integrals.

(a) $\displaystyle\int \left(e^{3x} + \frac{3}{1+(2x+1)^2} - 2\sec^2(5x) + 2\sin(\pi x) - \frac{4}{\sqrt{1-(x-1)^2}} \right) dx$

(b) $\displaystyle\int \frac{1 + 2\sqrt{x} + x + x^3}{x^3} dx$

Math 1210: Worksheet #9 **Fall 2017**

Name:	**A#:**	**Section:**

[**2**] 1. Suppose that $F(x) = x^x$ is an antiderivative of $f(x)$. Find $f(x)$.

[**6**] 2. Solve the initial value problem $y'' = \frac{x^2+\sqrt{x}}{x}$, $y(1) = 1$, $y'(1) = 0$.

[**3**] 3. If $F(x) = \displaystyle\int_{-\ln x}^{\sin x} e^{t^2}\,dt$, then compute $F'(x)$.

[**9**] 4. Compute the integral.

(a) $\displaystyle\int \frac{1}{\sqrt{2x-x^2}}dx$ (Hint: complete the square)

(b) $\displaystyle\int_0^1 \frac{e^{2x}-e^{-2x}}{e^{x+1}}dx$

(c) $\displaystyle\int_1^{\sqrt{3}} \frac{1}{3+x^2}dx$

Math 1210: Worksheet #10 **Fall 2017**

Name:	A#:	Section:

[3] 1. $\int_0^{\frac{\pi}{4}} \sin(x)\sec^6(x)dx$

[3] 2. $\int_0^{\ln(2)} \frac{e^{2x}}{e^{2x}+1}dx$

[3] 3. $\int_0^{\frac{1}{2}\ln(3)} \frac{e^{x}}{e^{2x}+1}dx$

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[3] 4. $\displaystyle\int_0^1 \frac{x^2-1}{\sqrt{1+x}}dx$

[4] 5. $\displaystyle\int \frac{x+2}{x^2+4}dx$

[4] 6. $\displaystyle\int \sqrt{1+\sqrt{x}}dx$