Name: SOLUTION	A#:	Section:

1. Compute the limit and explain why L'Hôpital's rule does not apply.

(a) $\lim_{x\to\frac{\pi}{4}} \frac{\sin x}{x} = \frac{\sin(\pi/4)}{\pi/4} = \frac{2\sqrt{2}}{\pi}$

L'Hôpital's rule does not apply because we do not have an indeterminate form.

(b) $\lim_{x\to\infty} \frac{\ln x + 3x + \cos x}{3\ln x + 3x + 3\cos x} = \lim_{x\to\infty} \frac{\frac{\ln x}{x} + 3 + \frac{\cos x}{x}}{3\frac{\ln x}{x} + 3 + \frac{3\cos x}{x}} = \frac{0+3+0}{0+3+0} = 1.$

$\lim_{x\to\infty} \frac{\frac{1}{x} + 3 - \sin x}{\frac{3}{x} + 3 - 3\sin x} = \lim_{x\to\infty} \frac{3-\sin x}{3-3\sin x}$ does not exist; so L'Hôpital's rule does not apply because it produces a limit that does not exist.

2. Compute the limit.

(a) $\lim_{x\to\frac{\pi}{2}} (2x-\pi)\sec x = \lim_{x\to\pi/2} \frac{2x-\pi}{\cos x} \overset{\text{L'Hôpital "}\frac{0}{0}\text{"}}{=} \lim_{x\to\pi/2} \frac{2}{-\sin x} = \frac{2}{-1} = \boxed{-2}$

(b) $\lim_{x\to 1} \frac{x - \ln x - 1}{x^2 - 2x + 1} \overset{\text{L'Hôpital "}\frac{0}{0}\text{"}}{=} \lim_{x\to 1} \frac{1-\frac{1}{x}}{2x-2} = \lim_{x\to 1} \frac{x-1}{2x(x-1)} = \lim_{x\to 1} \frac{1}{2x} = \boxed{\frac{1}{2}}$

(c) $\lim_{x\to 0^+} (\sin x)^x = e^{\lim_{x\to 0^+} \ln((\sin x)^x)} = e^0 = \boxed{1}$

$\ln((\sin x)^x) = x \ln(\sin x)$

$\lim_{x\to 0^+} x \ln(\sin x) = \lim_{x\to 0^+} \frac{\ln(\sin x)}{\frac{1}{x}} = \lim_{x\to 0^+} \frac{\frac{1}{\sin x}\cdot \cos x}{-\frac{1}{x^2}} = \lim_{x\to 0^+} -\frac{x^2 \cos x}{\sin x}$

L'Hôpital "$\frac{-\infty}{\infty}$"

$= \lim_{x\to 0^+} (-x)\cdot\frac{x}{\sin x}\cdot \cos x$

$= 0 \cdot 1 \cdot 1 = \boxed{0}$

(d) $\lim_{x\to 0} \frac{e^{2x} - 1 - 2x - 2x^2}{x\cos x - x} = \lim_{x\to 0} \frac{2e^{2x} - 2 - 4x}{\cos x - x\sin x - 1} = \lim_{x\to 0} \frac{4e^{2x} - 4}{-\sin x - \sin x - x\cos x}$

L'Hôpital "$\frac{0}{0}$" L'Hôpital "$\frac{0}{0}$"

$= \lim_{x\to 0} \frac{8e^{2x}}{-2\cos x - \cos x + x\sin x} = \frac{8e^0}{-3\cos(0) + 0\cdot\sin(0)} = \boxed{-\frac{8}{3}}$

L'Hôpital "$\frac{0}{0}$"

3. Prove that the functions $f(x) = \sec^2 x$ and $g(x) = \tan^2 x$ differ by a constant. (Hint: compute the derivatives.)

$f'(x) = 2\sec x \cdot \sec x \cdot \tan x = 2\sec^2 x \tan x$

$g'(x) = 2\tan x \cdot \sec^2 x$

$\therefore f'(x) = g'(x).$

4. Compute the indefinite integrals.

(a) $\int \left(e^{3x} + \frac{3}{1+(2x+1)^2} - 2\sec^2(5x) + 2\sin(\pi x) - \frac{4}{\sqrt{1-(x-1)^2}}\right) dx$

$= \frac{1}{3}e^{3x} + \frac{3}{2}\tan^{-1}(2x+1) - \frac{2}{5}\tan(5x) - \frac{2}{\pi}\cos(\pi x) - 4\sin^{-1}(x-1) + C$

(b) $\int \frac{1+2\sqrt{x}+x+x^3}{x^3} dx = \int (x^{-3} + 2x^{-\frac{5}{2}} + x^{-2} + 1)\, dx$

$= \frac{x^{-2}}{-2} + 2\cdot\frac{2}{3}x^{-\frac{3}{2}} + \frac{x^{-1}}{-1} + x + C.$