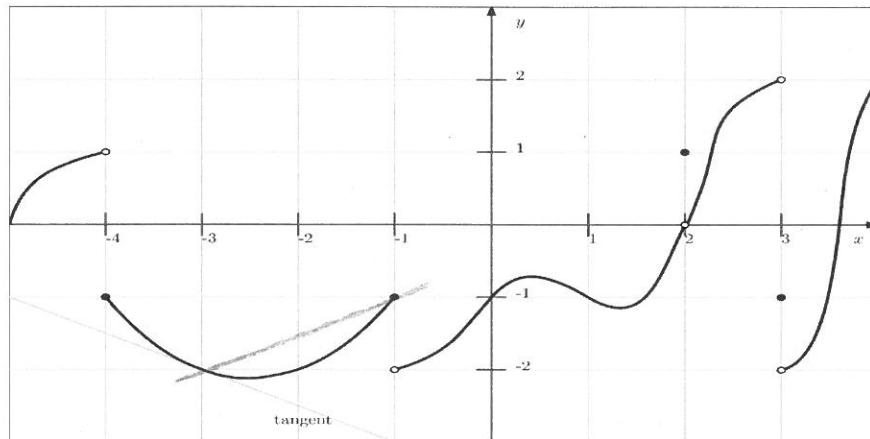


Name: **Solution**

A#:

Section: **I**

- [9] 1. Let f be a function whose graph of $y = f(x)$ is given below. Compute the following quantities or state that they do not exist.



(a) $f(3) = \underline{-1}$

(b) $\lim_{x \rightarrow 3^-} f(x) \underline{\lim_{x \rightarrow 3^-} f(x) = 2} \quad \& \quad \lim_{x \rightarrow 3^+} f(x) = -2 \Rightarrow \lim_{x \rightarrow 3} f(x) \text{ Does not exist}$

(c) $\lim_{x \rightarrow 2} (x^2 + 1 + f(x)) = \underline{4 + 1 + \lim_{x \rightarrow 2} f(x)} = 4 + 1 + 0 = 5$

(d) $\lim_{x \rightarrow 1^-} f(x) \underline{-1}$

(e) $\lim_{x \rightarrow -1^+} f(x) \underline{-2}$

(f) $\lim_{x \rightarrow -4^-} e^{x-1} f(x) \underline{\lim_{x \rightarrow -4^-} e^{x-1}} \lim_{x \rightarrow -4^-} f(x) = e^{-5} \times 1 = e^{-5}$

(g) The average rate of change of $f(x)$ over the interval $[-3, -1]$ $\frac{f(-1) - f(-3)}{-1 - (-3)} = \frac{-1 - (-2)}{-1 + 3} = \frac{1}{2}$

(h) The instantaneous rate of change of $f(x)$ when $x = -3$ $\underline{-1/2}$

(i) The equation of the secant line over the interval $[-3, -1]$ $\underline{y = \frac{1}{2}x - \frac{1}{2}}$

[3] 2. Let $f(x) = \begin{cases} x^2 - 1, & \text{if } x < 1 \\ e^x, & \text{if } x \geq 1 \end{cases}$. Then

$$(a) \lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1^-} x^2 - 1 = 0$$

$$(b) \lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} e^x = e^2$$

$$(c) \text{ The average rate of change of } f \text{ over the interval } [1, 3] \text{ is } \frac{e^3 - e^1}{3 - 1} = \frac{e^3 - e}{2}$$

[8] 3. Compute the limit or state that it does not exist.

$$(a) \lim_{x \rightarrow -2} \frac{x+2}{\sqrt{x^2-x-2}-2} \times \frac{\sqrt{x^2-x-2}+2}{\sqrt{x^2-x-2}+2} = \lim_{x \rightarrow -2} \frac{(x+2)(\sqrt{x^2-x-2}+2)}{\cancel{x^2-x-2-4}} \\ = \lim_{x \rightarrow -2} \frac{(x+2)(\sqrt{x^2-x-2}+2)}{(x+2)(x-3)} = \frac{\sqrt{4+2-2}+2}{-2-3} = \frac{4}{-5}$$

$$(b) \lim_{x \rightarrow -2^-} \frac{|x+2|}{x^2+5x+6} = \lim_{x \rightarrow -2^-} \frac{-(x+2)}{(x+2)(x+3)} = \frac{-1}{-2+3} = -1$$