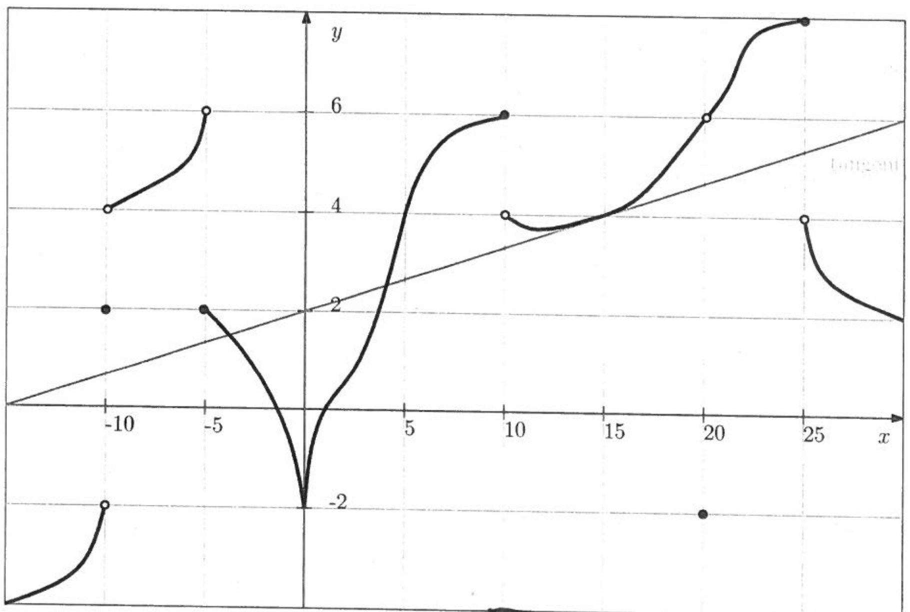
Math 1210: Worksheet #2 **Fall 2017**

Name: SOLUTION	A#:	Section: A, B

[2] 1. State the formal (ε, δ) definition of $\lim_{x \to a} f(x) = L$. Part marks will be given for an informal definition.

For every $\varepsilon > 0$ there is $\delta > 0$ such that for $|x-a| < \delta$ we have $|f(x) - L| < \varepsilon$

[5] 2. Let f be a function whose graph of $y = f(x)$ is given below.



(a) $\lim_{x \to 15} \frac{f(x)}{x^2 - 10x} = \frac{4}{(15)^2 - (10)(15)} = \boxed{\frac{4}{75}}$

(b) $\lim_{x \to 10^+} (2\ln(x) - 2f(x)) = (2\ln(10)) - 2(4) = \boxed{2(\ln 10) - 8}$

(c) List all values of a in the interval $(-15, 30)$ for which limit $\lim_{x \to a} f(x)$ does not exist:

$-10, -5, 10, 25$

(d) The average rate of change of $f(x)$ over the interval $[-5, 10]$ is $\frac{4}{15}$

(e) The instantaneous rate of change of $f(x)$ when $x = 15$ is $\frac{2}{15}$

[3] 3. Let $f(x) = \frac{1}{x}$ and let $a \neq 0$. Find the instantaneous rate of change of $f(x)$ when $x = a$, as a function (call it g) of a (you are not allowed to use any theory of derivatives).

$g(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{x \to a} \frac{\frac{1}{x} - \frac{1}{a}}{x - a} = \lim_{x \to a} \frac{\frac{a - x}{ax}}{x - a}$

$= \lim_{x \to a} \frac{(a - x)}{(x - a) \cdot ax} = \lim_{x \to a} \left(-\frac{1}{ax}\right) = \boxed{-\frac{1}{a^2}}$

v2.AB

4. Compute the limit or show that it does not exist.

[3] (a) $\lim_{x\to 2} \frac{\sqrt{x^2+5}-\sqrt{5x-1}}{x^2+3x-10} = \lim_{x\to 2} \frac{(\sqrt{x^2+5}-\sqrt{5x-1})(\sqrt{x^2+5}+\sqrt{5x-1})}{(x-2)(x+5)(\sqrt{x^2+5}+\sqrt{5x-1})}$

can skip

$= \lim_{x\to 2} \frac{(x^2+5)-(5x-1)}{(x-2)(x+5)(\sqrt{x^2+5}+\sqrt{5x-1})} = \lim_{x\to 2} \frac{x^2-5x+6}{(x-2)(x+5)(\sqrt{x^2+5}+\sqrt{5x-1})}$

$= \lim_{x\to 2} \frac{(x-2)(x-3)}{(x-2)(x+5)(\sqrt{x^2+5}+\sqrt{5x-1})} = \lim_{x\to 2} \frac{x-3}{(x+5)(\sqrt{x^2+5}+\sqrt{5x-1})}$

can. skip

$= \frac{2-3}{(2+5)(\sqrt{2^2+5}+\sqrt{5(2)-1})} = \frac{-1}{(7)(6)} = \boxed{-\frac{1}{42}}$

[3] (b) $\lim_{x\to 2^-} \left(\ln(2-x)-\ln(6-x-x^2)\right) = \lim_{x\to 2^-} \ln\left(\frac{2-x}{6-x-x^2}\right) = \lim_{x\to 2^-} \ln\left(\frac{2-x}{(2-x)(3+x)}\right)$

$= \lim_{x\to 2^-} \ln\left(\frac{1}{3+x}\right) = \ln\left(\frac{1}{3+2}\right) = \ln\left(\frac{1}{5}\right) = \boxed{-\ln 5}$

[4] (c) $\lim_{x\to 2} \frac{|4-x^2|}{x^2-x-2}$ d.n.e.

$\lim_{x\to 2^-} \frac{|4-x^2|}{x^2-x-2} = \lim_{x\to 2^-} \frac{4-x^2}{x^2-x-2} = \lim_{x\to 2^-} \frac{(2-x)(2+x)}{(x-2)(x+1)} = \lim_{x\to 2^-} \frac{-(2+x)}{x+1} = \boxed{-\frac{4}{3}}$

$\lim_{x\to 2^+} \frac{|4-x^2|}{x^2-x-2} = \lim_{x\to 2^+} \frac{-(4-x^2)}{x^2-x-2} = \lim_{x\to 2^+} \frac{-(2-x)(2+x)}{(x-2)(x+1)} = \lim_{x\to 2^+} \frac{+(2+x)}{x+1} = \boxed{+\frac{4}{3}}$