Math 1210: Worksheet #2 **Fall 2017**

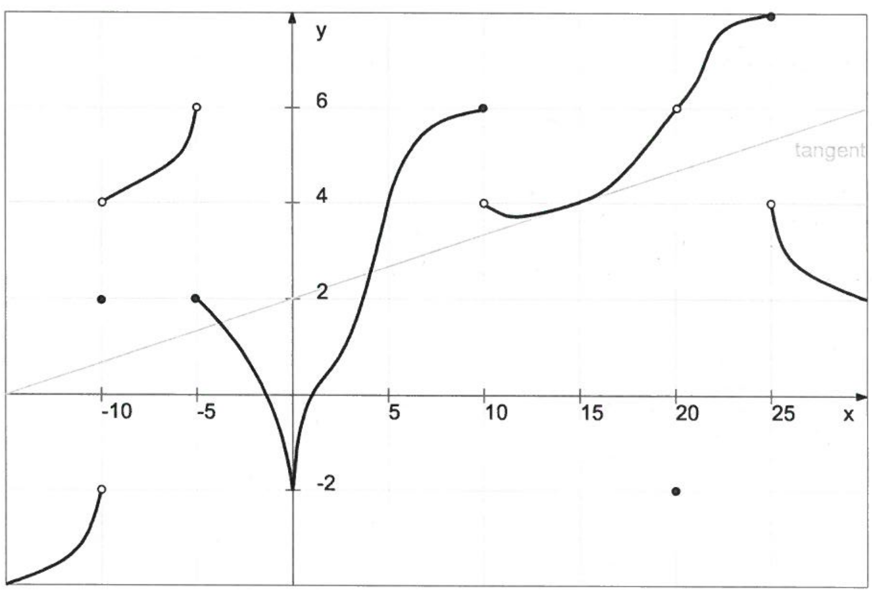
Name: ANS CD	A#:	Section:

[2] 1. State the formal (ε, δ) definition of $\lim_{x\to a} f(x) = L$. Part marks will be given for an informal definition. $(\forall \varepsilon > 0)(\exists \delta > 0)$ s.t. $|x-a|<\delta \Rightarrow |f(x)-L|<\varepsilon$

or "For all $\varepsilon > 0$ there exists $\delta > 0$ such that if ~~x~~ is within δ of a, $f(x)$ is within ε of L".

Informal: use judgement!

[5] 2. Let f be a function whose graph of $y = f(x)$ is given below.



(a) $\lim_{x\to 15} \frac{2f(x)}{x^2 - 10x} =$ $\frac{8}{75}$

(b) $\lim_{x\to 10^+} (\ln(x) - f(x)) =$ $\ln(10) - 4$

(c) List all values of a in the interval $(-15, 30)$ for which limit $\lim_{x\to a} f(x)$ does not exist:

$-10, -5, 10, 25$

(d) The average rate of change of $f(x)$ over the interval $[-5, 5]$ (note the differing scales of the axes!) is $1/5$

(e) The instantaneous rate of change of $f(x)$ when $x = 15$ is $2/15$

[3] 3. Let $f(x) = \frac{1}{x}$ and let $a \neq 0$. Find the instantaneous rate of change of $f(x)$ when $x = a$, as a function (call it g) of a (leave it as a limit; you are not allowed to use any theory of derivatives).

$$g(a) = \lim_{h\to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h}$$

v2.CD

4. Compute the limit or show that it does not exist.

[3] (a) $\lim_{x\to 2} \dfrac{\sqrt{x^2+5}-\sqrt{5x-1}}{x^2+3x-10}$

$$= \lim_{X\to 2} \frac{\sqrt{X^2+5}-\sqrt{5X-1}}{X^2+3X-10} \cdot \frac{\sqrt{X^2+5}+\sqrt{5X-1}}{\sqrt{X^2+5}+\sqrt{5X-1}}$$

$$= \lim_{X\to 2} \frac{X^2-5X+6}{X^2+3X-10\left(\sqrt{X^2+5}+\sqrt{5X-1}\right)}$$

$$= \lim_{X\to 2} \frac{\cancel{(X-2)}(X-3)}{\cancel{(X-2)}(X+5)\left(\sqrt{X^2+5}+\sqrt{5X-1}\right)}$$

$$= \frac{-1}{7(3+3)} = \boxed{\frac{-1}{42}}$$

[3] (b) $\lim_{x\to 2^-} \left(\ln(2-x)-\ln(6-x-x^2)\right)$

$$= \lim_{X\to 2^-} \ln\left(\frac{2-X}{(2-X)(3+X)}\right)$$

$$= \lim_{X\to 2^-} \ln(3+X) = \boxed{\ln 5}$$

[4] (c) $\lim_{x\to 2} \dfrac{|4-x^2|}{x^2-x-2}$

$$\lim_{X\to 2^-} \frac{|4-X^2|}{X^2-X-2} = \lim_{X\to 2^-} \frac{4-X^2}{X^2-X-2} = \lim_{X\to 2^-} \frac{(2-X)(2+X)}{(X+1)(X-2)} = -\frac{2+2}{2+1} = -\frac{4}{3}$$

$$\lim_{X\to 2^+} \frac{|4-X^2|}{X^2-X-2} = \lim_{X\to 2^+} \frac{X^2-4}{X^2-X-2} = \lim_{X\to 2^+} \frac{(X+2)(X-2)}{(X+1)(X-2)} = \frac{4}{3}$$

unequal, limit does not exist.

v2.CD