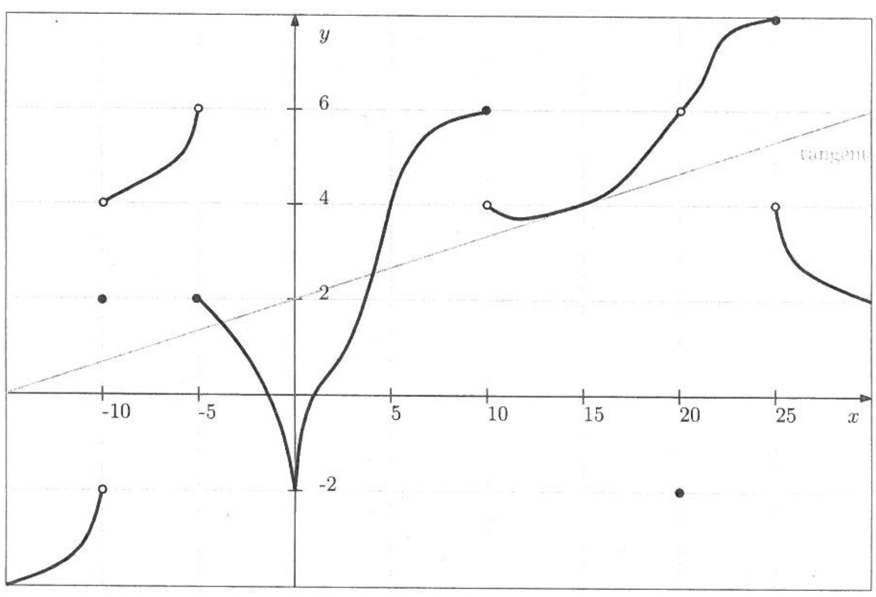
Math 1210: Worksheet #2 Fall 2017

Name: Solution	A#:	Section: E,F

[2] 1. State the formal (ε, δ) definition of $\lim_{x\to a} f(x) = L$. Part marks will be given for an informal definition.

Assuming that $f(x)$ exists for all x in some open interval containing a except possibly at a. $\lim_{x\to a} f(x) = L$ if for any number $\varepsilon > 0$ there is a corresponding number $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x-a| < \delta$

[5] 2. Let f be a function whose graph of $y = f(x)$ is given below.



(a) $\lim_{x\to 15} \dfrac{f(x)}{x^2+x-1} = \dfrac{\lim_{x\to 15} f(x)}{\lim_{x\to 15}(x^2+x-1)} = \dfrac{4}{225+15-1} = \dfrac{4}{239}$

(b) $\lim_{x\to 10^+} (2\ln(x+1) - 3f(x)) = 2\ln(11) - 3\lim_{x\to 10^+} f(x) = 2\ln(11) - 3\times 4$

(c) List all values of a in the interval $(-15, 30)$ for which limit $\lim_{x\to a} f(x)$ does not exist:

$-10, -5, 10, 25$

(d) The average rate of change of $f(x)$ over the interval $[-5, 10]$ is $\dfrac{f(10)-f(-5)}{10-(-5)} = \dfrac{6-2}{15} = \dfrac{4}{15}$

(e) The instantaneous rate of change of $f(x)$ when $x = 15$ is $1/3$

[3] 3. Let $f(x) = \frac{1}{x}$ and let $a \neq 0$. Find the instantaneous rate of change of $f(x)$ when $x = a$, as a function (call it g) of a (you are not allowed to use any theory of derivatives).

$$g(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a} = \lim_{x\to a} \frac{1/x - 1/a}{x-a} = \lim_{x\to a} \frac{\frac{a-x}{ax}}{x-a} = \lim_{x\to a} \frac{a-x}{ax(x-a)}$$

$$= \lim_{x\to a} \frac{-1}{ax} = \frac{-1}{a^2}$$

$$g(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h} = \lim_{h\to 0} \frac{\frac{1}{a+h} - \frac{1}{a}}{h} = \lim_{h\to 0} \frac{\frac{a-(a+h)}{a(a+h)}}{h} = \lim_{h\to 0} \frac{\frac{-h}{a(a+h)}}{h}$$

$$= \lim_{h\to 0} \frac{-\cancel{h}}{\cancel{h}a(a+h)} = \lim_{h\to 0} \frac{-1}{a(a+h)} = \frac{-1}{a^2}$$

4. Compute the limit or show that it does not exist.

[3] (a) $\lim_{x\to 3} \frac{\sqrt{x^2-2}-\sqrt{10-x}}{x^2-x-6} \times \frac{\sqrt{x^2-2}+\sqrt{10-x}}{\sqrt{x^2-2}+\sqrt{10-x}} = \lim_{x\to 3} \frac{\overbrace{(x^2-2)-(10-x)}^{x^2+x-12}}{(x^2-x-6)(\sqrt{x^2-2}+\sqrt{10-x})}$

$= \lim_{x\to 3} \frac{\cancel{(x-3)}(x+4)}{\cancel{(x-3)}(x+2)(\sqrt{x^2-2}+\sqrt{10-x})} = \frac{3+4}{(3+2)(\sqrt{7}+\sqrt{7})} = \frac{7}{10\sqrt{7}} = \frac{\sqrt{7}}{10}$

[3] (b) $\lim_{x\to -1^+} \left(\ln(1+x)-\ln(x^2+4x+3)\right) = \lim_{x\to -1^+} \left(\ln\left(\frac{1+x}{x^2+4x+3}\right)\right) =$

$\lim_{x\to -1^+} \left(\ln\left(\frac{\cancel{1+x}}{\cancel{(1+x)}(x+3)}\right)\right) = \ln\left(\frac{1}{2}\right) = -\ln(2)$

[4] (c) $\lim_{x\to 2} \frac{|4-x^2|}{x^2-x-2}$

$\lim_{x\to 2^+} \frac{x^2-4}{(x-2)(x+1)} = \lim_{x\to 2^+} \frac{\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)} = \frac{4}{3}$

$\lim_{x\to 2^-} \frac{4-x^2}{(x-2)(x+1)} = \lim_{x\to 2^-} \frac{-\cancel{(x-2)}(x+2)}{\cancel{(x-2)}(x+1)} = \frac{-4}{3}$

$\Longrightarrow \lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x) \Longrightarrow$ Does not exist