Math 1210: Worksheet #2 **Fall 2017**

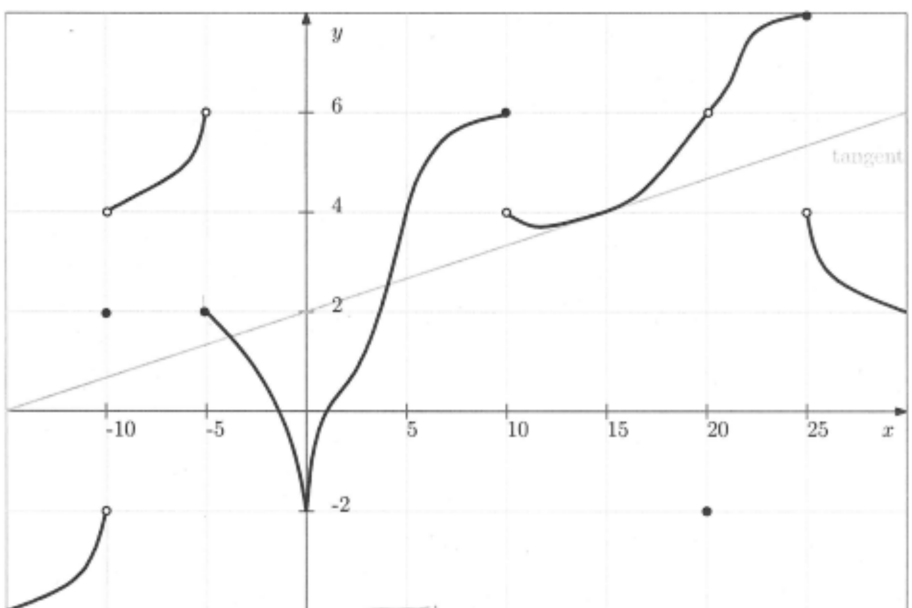
Name: Solutions	A#:	Section: G

[2] 1. State the formal (ε, δ) definition of $\lim_{x \to a} f(x) = L$. Part marks will be given for an informal definition.

Informal (not full marks): "$f(x)$ gets closer & closer to L as x gets closer & closer to a from both sides of a"

Formal (full marks): "for any $\varepsilon > 0$ there is a corresponding $\delta > 0$ such that $|f(x) - L| < \varepsilon$ whenever $0 < |x - a| < \delta$"

[5] 2. Let f be a function whose graph of $y = f(x)$ is given below.



(a) $\lim_{x \to 15} \dfrac{\overset{\nearrow 4}{f(x)}}{\underbrace{x^2 - 10x}_{\searrow 15(15-10)}} = \dfrac{4}{15 \cdot 5} = \boxed{\dfrac{4}{75}}$

(b) List all values of a in the interval $(-15, 30)$ for which $\lim_{x \to a^-} f(x) \neq f(a)$: $-10, -5, 20$

(c) List all values of a in the interval $(-15, 30)$ for which limit $\lim_{x \to a} f(x)$ does not exist:

$-10, -5, 10, 25$

(d) The average rate of change of $f(x)$ over the interval $[-5, 10]$ is $\dfrac{2-6}{-5-10} = \boxed{\dfrac{4}{15}}$

(e) The instantaneous rate of change of $f(x)$ when $x = 15$ is $\boxed{\dfrac{2}{15}}$

[3] 3. Let $f(x) = \sqrt{x-1}$ and let $a > 1$. Find the instantaneous rate of change of $f(x)$ when $x = a$, as a function (call it g) of a (you are not allowed to use any theory of derivatives).

$g(a) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \dfrac{\sqrt{x+h-1} - \sqrt{x-1}}{h}$

$= \lim_{h \to 0} \dfrac{1}{\sqrt{x+h-1} + \sqrt{x-1}}$

$= \boxed{\dfrac{1}{2\sqrt{x-1}}}$

$\dfrac{\sqrt{x+h-1} - \sqrt{x-1}}{h} \cdot \dfrac{\sqrt{x+h-1} + \sqrt{x-1}}{\sqrt{x+h-1} + \sqrt{x-1}}$

$= \dfrac{(x+h-1) - (x-1)}{h(\sqrt{x+h-1} + \sqrt{x-1})}$

$= \dfrac{1}{\sqrt{x+h-1} + \sqrt{x-1}}$

4. Compute the limit or show that it does not exist.

[3] (a) $\lim\limits_{x\to1^-}\dfrac{\sqrt{1+x}-\sqrt{x^2+1}}{x^2-5x+4}$

$$\frac{\sqrt{1+x}-\sqrt{x^2+1}}{(x-1)(x-4)}\cdot\frac{\sqrt{1+x}+\sqrt{x^2+1}}{\sqrt{1+x}+\sqrt{x^2+1}}$$

$$=\frac{(1+x)-(x^2+1)}{(x-1)(x-4)\left(\sqrt{1+x}+\sqrt{x^2+1}\right)}$$

$$=\frac{-x\cancel{(x-1)}}{\cancel{(x-1)}(x-4)\left(\sqrt{1+x}+\sqrt{x^2+1}\right)}$$

$$=\frac{-x}{(x-4)\left(\sqrt{1+x}+\sqrt{x^2+1}\right)}$$

$$=\lim_{x\to1^-}\frac{\overset{\nearrow -1}{-x}}{\underset{\downarrow\, -3}{(x-4)}\underset{\downarrow\, 2\sqrt{2}}{\left(\sqrt{1+x}+\sqrt{x^2+1}\right)}}$$

$$=\frac{-1}{-6\sqrt{2}}$$

$$=\boxed{\frac{1}{6\sqrt{2}}}$$

[3] (b) $\lim\limits_{x\to3^-}\left(\ln(3-x)-\ln(6+x-x^2)\right)$

$$\ln(3-x)-\ln(6+x-x^2)$$

$$=\ln\left(\frac{3-x}{6+x-x^2}\right)$$

$$=\ln\left(\frac{3-x}{-(x-3)(x+2)}\right)$$

$$=\ln\left(\frac{1}{(x+2)}\right)$$

$$=\lim_{x\to3^-}\ln\left(\frac{1}{x+2}\right)$$

$$\boxed{\begin{aligned}&=\ln\left(\tfrac{1}{5}\right)\\&=-\ln(5)\end{aligned}}$$

[4] (c) $\lim\limits_{x\to1}\dfrac{x^2-1}{|1-x|}$

$$|1-x|=\begin{cases}1-x & \text{if } x\le 1\\ x-1 & \text{if } x>1\end{cases}$$

We need to take one-sided limits and compare.

$$\lim_{x\to1^-}\frac{x^2-1}{|1-x|}=\lim_{x\to1^-}\frac{x^2-1}{1-x}=\lim_{x\to1^-}\frac{\cancel{(x-1)}(x+1)}{-\cancel{(x-1)}}\;\boxed{=-2}$$

$$\lim_{x\to1^+}\frac{x^2+1}{|1-x|}=\lim_{x\to1^+}\frac{x^2-1}{x-1}=\lim_{x\to1^+}\frac{\cancel{(x-1)}(x+1)}{\cancel{x-1}}\;\boxed{=2}$$

Since $\lim\limits_{x\to1^+}\dfrac{x^2-1}{|1-x|}\neq\lim\limits_{x\to1^-}\dfrac{x^2-1}{|1-x|}$ we see

that $\lim\limits_{x\to1}\dfrac{x^2-1}{|1-x|}$ D.N.E.

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